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ABSTRACT

Mathematics units developed during a summer workshop are presented. The purposes of the workshop were to prepare qualified secondary teachers to teach mathematics to low achievers and to collect, review, and develop new methodologies and materials for teaching the reluctant learner in mathematics. The units developed were designed to be used as supplementary materials and it was indicated that individual teachers should feel free to adapt the units to fit local needs. The units were designed to help students achieve needed competencies that were suggested by a committee of the National Council of Teachers of Mathematics. Each unit includes a competency statement, instructional objectives, and two suggested activities. Twenty-seven competencies are listed. The number of instructional objectives for each competency varies from one to fourteen. Some of the suggested competencies are: (1) ability to perceive patterns displayed by means of sequences of specific instances; (2) use the standard algorithms for the operations of arithmetic of whole rational numbers; and (3) construct bisectors of lines and angles. (MK)

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Activity Oriented Materials Developed to Help the Low Achiever Attain Basic Mathematical Competencies

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Prepared by
Members of an NSF Sponsored
Summer Conference
In Attacking the Problem of Teaching Mathematics
to the Low Achiever in Secondary Schools
Summer 1971

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SE 029 385

Introduction

Explanation of Workshop

A three-week summer workshop in attacking the problem of teaching mathematics to the reluctant learner in secondary schools was held at the University of Nebraska, Lincoln, Nebraska, during the summer of 1971 under the direction of Milton W. Beckmann, Supervisor of Mathematics and Professor of Secondary Education and Jack L. Beal, Graduate Assistant.

Purpose of the Workshop

The purpose of the workshop was to:

(1) better prepare qualified secondary teachers to teach mathematics to low achievers

(2) collect, review, and develop new methodologies and materials for teaching the reluctant learner in mathematics

Units Developed

a. Competencies considered

Some time ago the directors of the National Council of Teachers of Mathematics created a Commission. Its assignment was to decide the basic mathematics needed for personal use by every citizen....not just for those who planned to go to college or for those engaged in technical professions and trades. Just recently the directors of the National Council of Teachers of Mathematics selected a Committee on Basic Mathematical Competencies. It is listing what is believed to be the basic minimum mathematical competencies and skills needed by enlightened citizens. We received the tentative list. The 29 competencies seem to form a subset of the more recently proposed competencies. The units were written to help students achieve these competencies.

b. Units to be used as Supplementary Materials

These units are to be used to supplement the teacher's regular presentations and are not meant to be self-contained learning packets. The units were designed for but not exclusively for students who score below the 30 percentile on a standardized achievement test.

c. Changes to made for Local Conditions

Realizing that all districts operate under different conditions, the teacher should feel free to adapt these materials to his own students.

d. Organizational Pattern

Units were organized around the following pattern.

Competency
Instructional Objectives
Activity 1
    Suggested materials
    Directions to the student
    Suggested strategies
Activity 2
    etc.

Attainment of the listed instructional objectives for any one unit indicates the student possesses part if not all of the competency. The teacher may find it necessary to use more than one unit in order to help the student achieve the entire competency. The instructional objectives will help the teacher determine that part of the competency taught by the activities in the unit. It should be possible to use one or all of the activities in a unit independent of each other. For example, if Activity 1 requires some materials that the teacher does not possess, then he may choose to use Activity 2.

The directions to the students should be sufficiently clear to be used by individual students with minimal directions from the teacher. In fact we suggest that the teacher write the directions to the students on 5 x 8 cards.

Under suggested strategies, the suggestion that students work in small or large groups may be found. It was not our intent that all activities need to be done individually. Therefore, we have also included small and large group work.

Final Editing Is Not Complete.

Participants of the workshop felt that these units would prove exceedingly helpful to their students. Time was a factor due to the large number of units that were developed, consequently final editing is not complete for all units.

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Competency/Objective — Page

1. Ability to perceive patterns displayed by means of sequences of specific instances.
   - a. Identify Fibonacci Sequence. 1 - 4
   - b. Identify interesting properties of Fibonacci Sequence. 1 - 4
   - c. Given a sequence of events that illustrate a pattern, the student will be able to make a generalization describing the pattern. 5 - 11

2. Describe a given positive rational number using decimal, percent, or fractional notation.
   - a. The student will be able to change a percent to its decimal equivalent. 12 - 19

3. Write an equivalent fraction for given fractions, such as 1/2, 2/3, and 3/5.
   - a. Given a fraction not in lowest terms, the student will be able to write the equivalent fraction in lowest terms. 20 - 26
   - b. Given a fraction, the student will be able to write at least one equivalent fraction. 20 - 26

4. Use the standard algorithms for the operations of arithmetic of whole rational numbers.
   - a. The student will be able to add single digit numbers. 27 - 30
   - b. The student will be able to designate the addends and sum. 27 - 30
   - c. The student will be able to state addition facts of numerals 1 - 10. 31 - 34
   - d. The student will be able to state subtraction facts of one digit numerals less than 20. 31 - 34

e. The student will find the product of any two numbers between 5 and 10, given the multiplication facts up to 5 x 5 and the addition facts. 35 - 39

f. Given a simple multiplication problem the student will be able to give an inverse relationship and find the answer. 40 - 48

g. Given a simple division problem, the student will be able to give an inverse relationship and find the answer. 40 - 48

h. Given a simple division problem in which the divisor and dividend are only one number, the student can find the quotient. 49 - 51

i. Given a division problem in one of three forms, the student can recognize which member of the problem is the divisor, and which is the dividend. 52 - 54

j. Given a division problem in which the divisor is a one digit number and which has no remainder, the student can find the quotient. 55 - 64

k. Given a division problem in which the divisor is a one digit number and which has a remainder, the student can find the quotient. 65 - 72

l. Given a division problem with a two digit divisor--units digit less than 5, the student can find an answer. 73 - 80

m. Given a division problem with a two digit divisor--units digit equals 5, the student can find an answer. 81 - 82

n. Given a division problem with a two digit divisor--units digit greater than 5, the student will find an answer. 83 - 86

5. Solve addition, subtraction, multiplication and division problems with fractions having denominators less than twenty.

a. Given a fractional addition or subtraction problem having denominator less than 20, the student will be able to solve it. 87 - 92

  - b. Given any combination of fractions, mixed numbers, and whole numbers to perform the student can perform the required subtraction. 93 - 95

6. Solve additon, subtraction, multiplication and division problems with decimal fractions.
  - a. Given any combination of decimals and whole numbers, the student can perform the required subtraction. 96 - 98

7. Construct a true mathematical statement from a given verbal problem.
  - a. The student will recognize mathematical phrases and be able to translate verbal phrases into those involving symbols and vice versa. Student will give examples of mathematical phrases orally and in writing. The student will produce phrases using symbols, having been given word phrases and vice versa. 99 - 100
  - b. The student will construct mathematical sentences using phrases combined with symbols for equality and inequality. 101 - 102
  - c. The student will be able to write open sentences given certain conditions to be met. Given an open sentence, the student will be able to make up a verbal problem to fit it. 103

8. Solve simple linear equations.
  - a. The student will transform open sentences into true sentences without using the axioms. 104 - 105
  - b. The student will find an operation which undoes a given one. 106 - 107
  - c. The students will be able to use the multiplication axiom in solving equations. 108 - 109
  - d. The student will be able to solve open sentences through use of the axioms. 110

9. Construct a graph indicating the relationship of two variables from a given set of data.

    a. Given a grid with a horizontal scale and a vertical scale, the student should be able to locate on this grid points corresponding to an ordered pair. 111 - 117

    b. Given a point on a grid, the student should be able to write the ordered pair that describes the location of this point. 118 - 120

    c. Given a grid and a set of ordered pairs the student should be able to draw the design formed by the points located by the ordered pairs. 121 - 125

10. Describe parallel lines, perpendicular lines and intersecting lines using drawings or intuitive concepts.

    a. The student can recognize a line segment from drawings and objects in his surroundings. 126 - 127

    b. The student can recognize and describe parallel lines from drawings and objects in his surroundings. 128 - 130

    c. The student will be able to recognize intersecting lines. 131 - 132

    d. The student will be able to recognize perpendicular lines. 133 - 136

    e. The student will be able to classify lines as parallel, perpendicular or intersecting. 137 - 138

11. Classify simple plane figures by distinguishing some of their properties.

    a. The student is able to group the triangles from a given set of triangles into subsets of equilateral isosceles, and scalene triangles; and he is able to name the respective subsets. 139 - 141

    b. The student is able to select the triangles and quadrilaterals from a set of models of polygons. 142 - 144

	c. The student will be able to find the measure of the third angle of a triangle when the measure of two of the angles are known.	145 - 146
12.	Compute the perimeter of a given polygon.	
	a. By use of measuring instruments the student should exhibit the ability to determine the perimeter of plane geometric polygons.	147
	b. The student demonstrates his ability to compute the perimeter of polygons through estimation and the use of standard units and by using a formula.	148 - 150
13.	Compute the area of a rectangle and of a triangle.	
	a. Given a rectangle and a triangle, the student will be able to compute the area of each figure.	151 - 153
14.	Construct bisectors of lines and angles.	
	a. Given an angle, the student will be able to divide the angle into two equal parts.	154 - 158
	b. Given a line segment, the student will be able to locate the midpoint.	159 - 162
15.	Identify the conditions for similarity of triangles and use the properties of similarity to solve problems.	
	a. The student will be able to recognize similar triangles and solve problems relating to similar triangles.	163 - 166
16.	Classify solid figures by distinguishing some of their properties.	
	a. The student will be able to select a cube from a set of geometric solids.	167
	b. The student will be able to identify the six congruent faces, the twelve straight line edges, and the eight vertices of a cube.	167

c. The student will be able to determine the two-dimensional patterns that can be used to construct a cube, and will construct one or more cubes from these patterns. 168 - 171

17. Apply the common English measures of length, volume, weight, time, money, and temperature.

a. The student will be able to find the length of the edges, the areas of the faces, and the volumes of selected cubes. 172 - 173

b. Given an object to be measured, the student can give its length--accurate to the nearest unit of measure. 174 - 175

c. Given an object to be weighed, the student can give its weight--accurate to the nearest unit of measure. 176

d. Given a surface to be measure, the student can give its area--accurate to the nearest unit of measure. 177

e. Give a rectangular container, the student will be able to measure it with standard cubic measure. 178

18. Convert from one measure to an equivalent one with larger or smaller units in the English System.

a. Having become acquainted with the English units of measurement, the student will determine relationships between these units. 179 - 182

19. Convert, using tables, English to metric measure and conversely.

a. Given a measurement of length in the English or Metric Systems, the student will be able to convert to the other system. 183

b. Given a measurement of weight in the English or Metric System the student will be able to convert to the other system. 184

20. Recognize that no measurement is precise.

   a. Given a compass the student will determine which lines are longer than a given line. 185

   b. Given seven line segments, a group of students will each measure the items, accurate to the nearest 1/16 of an inch and nearest millimeter, record this information and then compare the results. 186

21. Use metric units of length, mass and volume in making measurements.

   a. Given a set of objects, or drawings of same, the student will measure, calculate, and report on volume expressed in metric units. 187 - 188

   b. Given a graduated milliliter measuring cup, the student will measure a milliliter, centiliter, and liter. 189

22. Use standard measuring devices of length, area, volume, time, and temperature to make measurements.

   a. Given a group of line segments, the students will measure them accurately to the nearest inch, 1/2 inch, 1/4 inch, and 1/8 inch. 190 - 192

   b. Given various scales, the student will weigh objects, record their results. 193

   c. Given thermometers of Fahrenheit and Celsius (Centigrade) scales, the student will take the temperature of six water samples. 194

   d. The student will be able to measure and draw a line segment using a ruler. 195 - 196

   e. The student will be able to draw and measure an angle of 90° using a protractor. 195 - 196

   f. The student will be able to draw and measure angles using a protractor. 197 - 199

23. Round off measurements to the nearest given unit of the measuring device used such as rulers, protractors and thermometers.

   a. Given various angles and geometric shapes, the student will use a protractor to measure the angles to the nearest degree. 200 - 202

   b. Given a map with a list of cities and their respective population the students will round off this figure to the nearest hundred. 203 - 205

   c. Given printed centigrade and fahrenheit thermometer scales, the student will round off comparable temperatures. 203 - 205

24. Predict the probability of simple events occuring.

   a. Through the use of simple devices the student should demonstrate and predict the probability of simple events occuring. 206 - 211

25. Plan a budget including record keeping of personal and travel expenses.

   a. Given the hourly rate and the number of hours worked during a week, the student should be able to determine the weekly and bi-weekly wages. 212 - 215

26. Use adders and/or calculators to solve addition, subtraction, multiplication, and division problems.

   a. To be able to add a series of one, two, three, or more eight digit numbers with an adding machine. 216

   b. To be able to subtract whole numbers and decimal numerals. 217 - 219

   c. To be able to multiply, where the multiplier is a one digit numeral with the adding machine. 220 - 223

27. Write simple sentences showing the relation =, <, >, for two given numbers.

   a. The student will be able to correctly select either =, <, or >, to make x _____ y a true statement for given values of x and y. 224 - 229

COMPETENCY: Ability to perceive patterns displayed by means of sequences of specific instances.

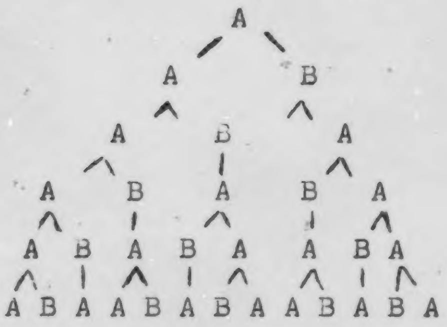
OBJECTIVES:
1. Identify Fibonacci Sequence.
2. Identify interesting properties of Fibonacci Sequence.

ACTIVITY 1: A teacher directed class activity to discover the Fibonacci Sequence.

Suggested Materials:

1. Historical Background: Fibonacci was an Itanian who lived about 700 years ago. He wrote one of the first books in Europe about Hindu-Arabic numbers which are our number symbols as we know them today. Fibonacci's book showed how the use of the Hindu-Arabic system simplified calculation. He invented many methods of computation and explained them in his book. Fibonacci had a problem in his book which has many interesting relations today. The problem asks: How many pairs of rabbits are born of one pair in a year?

2. Overhead or chalkboard space on which to plot the rabbit population problem and tally the number of rabbits, thus developing the Fibonacci Sequence. Let the class discover the pattern and continue the sequence.



Date	Pairs	Pairs of Adults (A)	Pairs of Babies (B)	Total Pairs
Jan 1	A	1	0	1
Feb 1	A B	1	1	2
Mar 1	A B A	2	1	3
Apr 1	A B A B A	3	2	5
May 1	A B A B A A B A	5	3	8
Jn 1	A B A A B A B A A B A B A	8	5	13
Jul 1	?	?	?	?
Aug 1	?	?	?	?
Sep 1	?	?	?	?
Oct 1	?	?	?	?
Nov 1	?	?	?	?
Dec 1	?	?	?	?

Directions for Students:

One of the great mathematicians of all times was Leonardo da Pisa, often nicknamed Fibonacci or son of nature. He made many exciting discoveries in mathematics that some of you might want to read more about. However, he is remembered mainly for a sequence of numbers: 1, 1, 2, 3 . . . . to which his name has been applied. The sequence is derived from a hypothetical problem stated in his book thus: "How many rabbits can be produced from a single pair in a year if it is supposed that every month each pair begets a new pair which from the second month becomes productive."

Suggested Strategies:

Let the class develop the sequence, making sure that everyone "discovers" the pattern. Build a sequence for the first 20 terms.

ACTIVITY 2: Perceive the pattern displayed by selecting a number from the Fibonacci sequence other than the first one. Square this number. Now multiply the numbers just before and after this number. How do your answers compare?

Suggested Materials:

Fibonacci sequence, paper, pencil.

Directions to Students:

Build a table such as this and enter 10 examples.

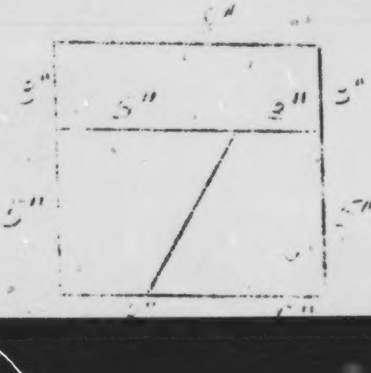
Number to be squared	Result of squaring	Numbers to be multiplied	Product	How the answers compare
5	25	3 x 8	24	24 + 1 = 25
8	64	5 x 13	65	65 - 1 = 64

ACTIVITY 3: Any triple of Fibonacci numbers such that the middle one has an even subscript will produce an absolute difference of one. i.e. $(5 \times 13) - 8^2 = -1$.

Suggested Materials:

Writing materials.

Oak tag square 8" x 8" measured off as follows:



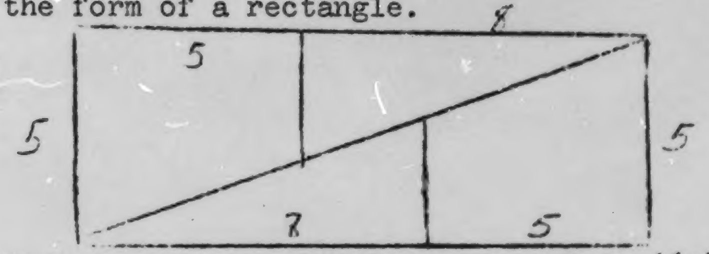
Directions for Students:

Below is the Fibonacci sequence in one to one correspondence with F and the proper subscript. Choose any Fibonacci number with an even subscript, square the number, then multiply the number before and after and the difference will be 1 or -1.

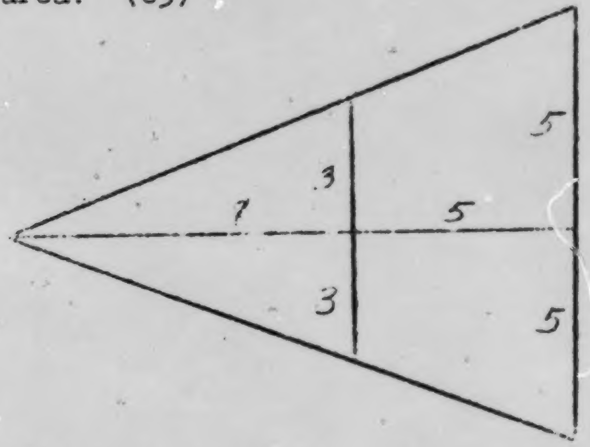
1, 1, 2, 3, 5, 8, 13, 21, 34

F_1 F_2 F_3 F_4 F_5 F_6 F_7 F_8 F_9

Look at your oak tag square. What is the area? Now cut it into 4 pieces and rearrange these pieces in the form of a rectangle.



What is the area of this rectangle? (65) Now arrange the pieces in the form of a triangle. What is this area? (65)



Suggested Strategies:

What other unit measures could you use to make this magic trick? Remember the 8 had an even subscript. Try 21.

ACTIVITY 4:

Suggested Materials:

Fibonacci sequence, paper, pencil.

Directions to Students:

Select a Fibonacci number. Square this number. Multiply the second number before and the second number after. Compare the two answers. Is there a pattern?

Build a table such as this and enter 10 examples.

Number to be squared	Result of squaring	Numbers to be multiplied	Product	How does square compare with product?
5	25	2 x 13	26	26 - 1 = 25
8	64	3 x 21	63	63 + 1 = 64

ACTIVITY 5: Any two consecutive Fibonacci numbers multiplied together will give the product of the numbers on either side by plus or minus 1.

Suggested Materials:
Writing materials. A sequence of Fibonacci numbers.

Directions for Student:
Build a table such as this and enter 10 examples.

Two consecutive Fibonacci numbers	Product	Numbers on either side	Product	How do products compare?
1, 2	2	1, 3	3	2 + 1 = 3
2, 3	6	1, 5	5	6 - 1 = 5
3, 5	15	2, 8	16	15 + 1 = 16
5, 8	40	3, 13	39	40 - 1 = 39

COMPETENCY: Ability to perceive patterns displayed by means of sequences of specific instances.

OBJECTIVE:

Given a sequence of events that illustrate a pattern, the student will be able to make a generalization describing the pattern.

ACTIVITY 1: Tangrams

Suggested Materials:
cardboard, construction paper or oak tag, glue, scissors, ditto copies of the student worksheet

Directions to student:
Follow the instructions on your worksheet.

Suggested strategies:
As a background for this activity, the teacher could tell the following story or it could be printed on the student's worksheet:
Tan, a Chinese nobleman, had a square ceramic tile he valued very highly. One day he dropped the tile, and it broke into seven pieces. Tan spent the rest of his life trying to put the tile back together. He never made the square, but he found many other shapes.

ACTIVITY 2: Pattern for multiplying by 11

Suggested materials:
ditto copies of student worksheet

Directions to student:
Follow the instructions on your worksheet.

Suggested strategies:
After the students understand the pattern illustrated on their worksheet, the teacher may want to direct them to modify the pattern. For example, where the sum of the digits in greater than 9 as in 57 x 11 = 627.
Also, this pattern could be extended to 3 or more digit numbers.

ACTIVITY 3: Sequences (small group or individual activity)

Suggested materials:
material to use for counters, this might be chips, beans kernels of corn, or pieces of paper; worksheets

Directions to student:
Follow the instructions on the worksheet.

ACTIVITY 1: Worksheet

Directions to the student:

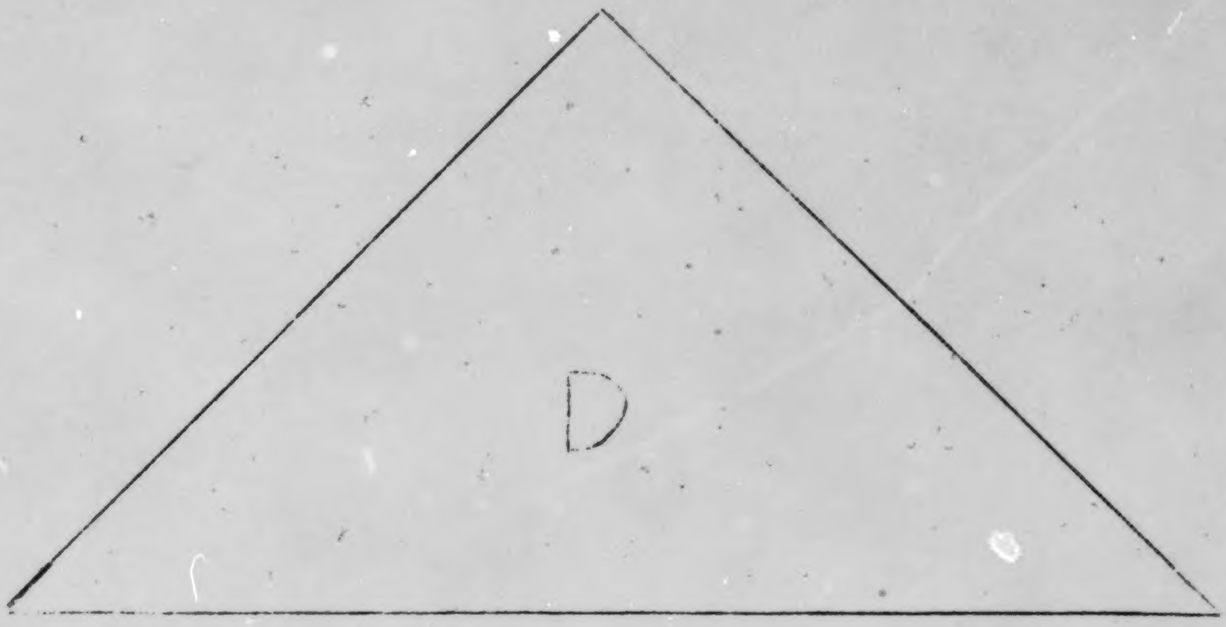
1. Cut around the outside of the large square from page 2 of the worksheet. Then glue it on the heavy paper given to you by your teacher. Cut the smaller numbered pieces apart. Practice making some designs with these pieces.

2. a. Try to find 2△'s that will make a shape like ▱ .

   b. Use 2 △ 's to make a □ .

   c. Use a □ and 2 △ 's to make another △ .

   d. Use all 7 pieces and make the large △ as shown on this worksheet.



   e. Now put your 7 tangram pieces back into a square.

ACTIVITY 4: Divisibility by 3 (teacher directed) Magic Game

Suggested materials:
Explanation of the Magic Game sheet

Directions to student:

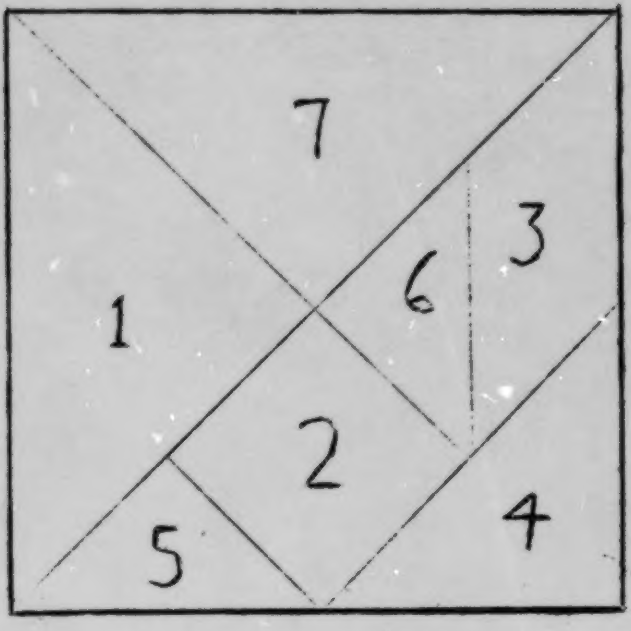
1. The teacher will ask you to give him a number. It can be any number.
2. The teacher will add one number to the digits and the new number will then be divisible by 3.
   He will add the number anywhere you want him to add it; before your number, in the middle or at the end.
3. The new number will be divisible by 3. Divide the new number by 3 to make sure the teacher didn't make a mistake.

Suggested strategies:

1. A good motivational idea would be to have the students try this game on their parents or friends.
2. When you use the word 'divisible', be sure that they understand the meaning of the word.

Activity 1: Worksheet

Part 1)



Part 2), (e)

ACTIVITY 2 Worksheet

Directions to the student:

This study will show how you can use patterns to help you multiply by 11 easily. Look at the following examples carefully. Notice the $\boxed{}$ and $\bigcirc$. They are trying to tell you something.

(a) $\begin{array}{r} 2\boxed{3} \\ \underline{\times\ 11} \\ 23 \\ \underline{23} \\ 25\boxed{3} \end{array}$

(b) $\begin{array}{r} \textcircled{3}5 \\ \underline{\times\ 11} \\ 35 \\ \underline{35} \\ \textcircled{3}85 \end{array}$

(c) $\begin{array}{r} \boxed{45} \\ \underline{\times\ 11} \\ 45 \\ \underline{45} \\ 4\textcircled{9}5 \end{array}$ +

In the following problems some of the work has not been written on the paper. Complete these problems by putting a number in the $\boxed{}$ or $\bigcirc$. Use the pattern of problems a to f.

(d) $\begin{array}{r} 3\boxed{3} \\ \underline{\times\ 11} \\ 33 \\ \underline{33} \\ 36\boxed{3} \end{array}$

(e) $\textcircled{8}1 \times 11 = \textcircled{8}91$

(f) $\begin{array}{r} \boxed{27} \\ \underline{\times\ 11} \\ 27 \\ \underline{27} \\ 2\boxed{}7 \end{array}$ +

(g) $\begin{array}{r} 7\boxed{2} \\ \underline{\times\ 11} \\ 72 \\ \underline{72} \\ 79\boxed{} \end{array}$

(h) $\begin{array}{r} \textcircled{6}1 \\ \underline{\times\ 11} \\ \bigcirc 71 \end{array}$

(i) $\boxed{53} \times 11 = 5\boxed{}3$ +

Answer these questions:

1. How did you know to put a 2 in $\boxed{}$ in (g)?

2. Do you see a pattern that will help you know what to put in the $\bigcirc$ for part (h)?

3. What different pattern tells you to put an 8 in the $\boxed{}$ in (f)?

4. Explain this pattern to your teacher or one of your friends.

5. Use your patterns to find these answers. Do not write out the work.
Example--

(a) $\begin{array}{r} 21 \\ \underline{\times\ 11} \\ 231 \end{array}$

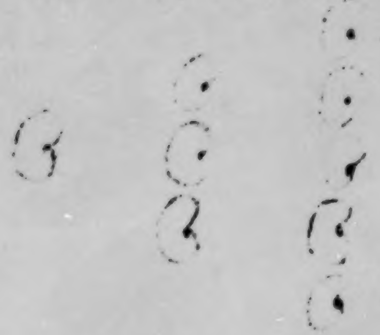
(b) $\begin{array}{r} 36 \\ \underline{\times\ 11} \end{array}$

(c) $41 \times 11 =$

(d) 52×11

ACTIVITY 3: Worksheet

A. Look at this list of numbers--1, 3, 5, 7, 9. On your work table make rows of beans to represent each number. Your work would start like this--



Continue your work on the table until you have a row for each number in the list. Try to represent the number that would come next after 9. What is it? What is special about the rows of beans? We would say this list of numbers is special because they follow a pattern. Do you see the pattern? Write the next three numbers in this list.

B. Look at this list of numbers--3, 7, 11, 15 . . . Use rows of beans to find the pattern. Write on your paper the next four numbers that would come in this list when the same pattern is followed.

C. Here is another list of numbers. Use rows of beans to find the pattern. (Notice that this is a different kind of pattern.)
0, 1, 1, 2, 3, 5, 8, 13, . . .
What number would come next in this list?

D. Here is another kind of pattern shown in this list. Can you find it? Use the same method we have been using.
1, 2, 4, 8, 16, . . .
Again, write the next four numbers in the list.

E. Here are some lists that follow some pattern. Where there is a box, a number has been left out of the list. See if you can use what you have learned to fill in the proper number in the boxes.

1. 2, 4, ☐ 8, ☐
2. 2, 5, 8, ☐ , 14
3. 3, 6, ☐ , 12, 15, ☐
4. 1, 4, 9, ☐ , 25
5. 3, 3, 3, ☐ , 3

ACTIVITY 4: Explanation of the Magic Game

Idea of the game: The students give a number and the teacher, using the rule for divisibility by 3, will add one digit (placed anywhere in the number) and make the number divisible by 3.

Rule of divisibility: A whole number is divisible by 3 if the sum of the digits is a multiple of 3. Therefore, when a student gives a number, the teacher adds the sums of the digits mentally and then adds whatever number is necessary to make the sum a multiple of 3.

Example of the game: The teacher explains the game to the students and has one student give a number. Suppose he gives 3421. The teacher adds a 2 anywhere in the sequence of digits (to make the sum of the digits divisible by 3) such as 34212. Then divide by 3, getting a quotient of 11404 remainder 0 which shows that it is divisible by 3.

The student may give a number that is already divisible by 3 such as 3945. In such a case you can say that it is already divisible by 3 or, to keep the game going, add 3 or 6 or some other multiple of 3 in the sequence of digits.

In each case by sure that either the teacher or the student carries through the checking of the solution to make sure the number is divisible by 3.

Continue this game and ask for volunteers who think they understand how you decide what numbers to add in the sequence of digits. Have them keep it a secret until as many as possible discover it on their own. Then the class or group could discuss how to check if a number is divisible by 3.

COMPETENCY: Describe a given positive rational number using decimal, percent, or fractional notation.

OBJECTIVES: The student will be able to change a percent to its decimal equivalent.

Examples:

(a) 64% = .64

(b) 5% = .05

(c) $6\frac{1}{2}\%$ = 6.5% = .065

(d) 147% = 1.47

ACTIVITY 1: Mathematical Bingo (See Instruction Sheet I)

Suggested materials:

pencil, paper, Mathematical Bingo cards, small pasteboard squares

Directions to student:

1. Place your Mathematical Bingo card face up on your desk.
2. Cover the center square marked "Free" with one of the small pasteboard squares.
3. The teacher will draw problems from container one at a time and write on the board or overhead projector.
4. Work each problem on your paper. (NOT on your desk and NOT on the Bingo card.)
5. If the answer appears on your Bingo card, cover it with a small pasteboard square. If it does not, wait for the next problem.
6. The first person to get a complete line of correct answers covered either up-and-down, across, or diagonally wins the game.

Suggested strategies:

1. Prior to introducing the game, the teacher should review the meaning of percent.

   Example: 64% = 64 ÷ 100 = 64/100 = .64

   3% = 3 ÷ 100 = 3/100 = .03

2. The teacher should discuss fractional and mixed number percents.

   Example: (a) ½% = ½ ÷ 100/1 = 1/200 = .005 and
   ½% = .5% = .005

   (b) 5 3/4% = 5.75 = .0575 and
   5 3/4% = 23/4% = 23/4 ÷ 100 = .0575

3. The teacher should explain percents greater than 100%

   Example: 163% = 1.63 100 = 1.00

4. Give cards to students a short while before explaining the game. This will arouse their curiosity and help to stimulate interest.

Instruction Sheet I

Mathematical Bingo

The game is played in a manner similar to regular bingo, but instead of calling a number under the letters B, I, N, G, O the teacher will draw and call a percent. Each pupil will change the percent to its decimal equivalent and cover the answer if it appears on his Mathematical Bingo card. To win, a student must have a complete row covered either horizontally, vertically, or diagonally. All students should keep their game boards intact until after the teacher has checked the answers of the pupil who has indicated that he has won. A sample Mathematical Bingo card and a sample Master Talley List (for the teacher's uso) appears below.

MATHEMATICAL		BINGO		
1.16	.423	1.00	.25	.73
.17	.06	.098	5.65	.10
.83	.23	FREE	.001	.0475
.79	.625	2.50	.03	.18
2.00	.09	.0625	.125	.34

Teacher's Tally List				
1%	22%	100%	.1%	½%
2%	23%	110%	43.3%	¼%
3%	25%	115%	62½%	.75%
4%	30%	125%	6¼%	7/8%
5%	34%	200%	5 3/8%	.003%
6%	38%	250%	4 3/4%	2.5%
7%	75%	463%	12½%	1/8%
8%	77%	518%	16 2/3%	1 1/8%
9%	79%	565%	33 1/3%	23.6%
10%	83%	1124%	37½%	8 1/3%
15%	88%	116%	.4%	4 5/8%
17%	90%	423%	.02%	56.5%
18%	96%	475%	.7%	4.75%

ACTIVITY 2: Play "Twenty-One." (See Instruction Sheet II.)

Suggested materials:

A sufficient number of decks of "Twenty-One" cards for the total number of students in the class.

Pencil and paper for making calculations, keeping score, keeping account of predictions, etc.

Directions to students:

See Instruction Sheet II.

Suggested strategies:

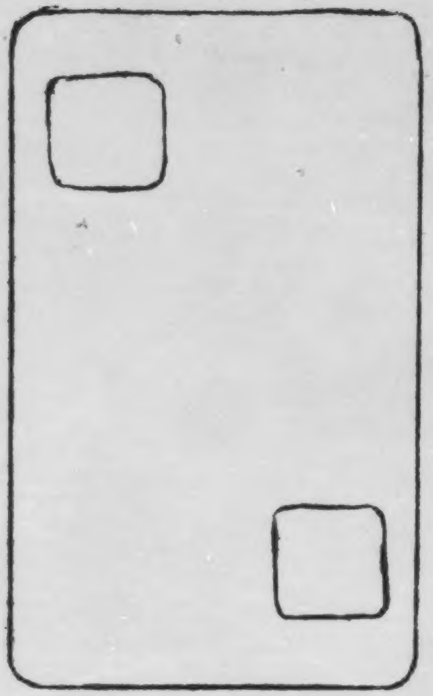
Same as for Activity 1.

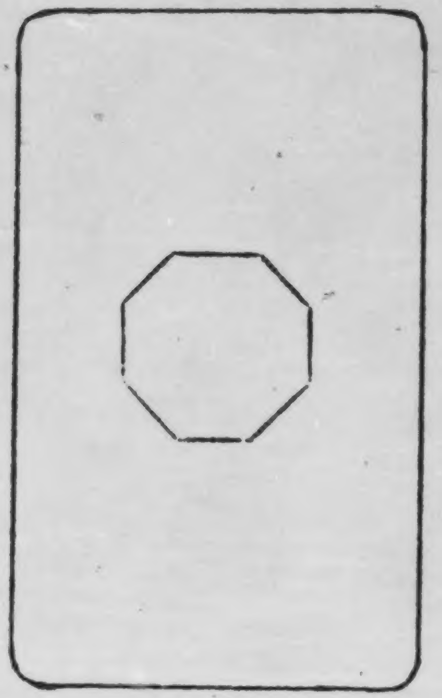
Instruction Sheet II

TWENTY-ONE

PURPOSE

This is a team game which can be used to provide drill in changing percents to their decimal equivalents.

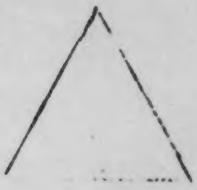


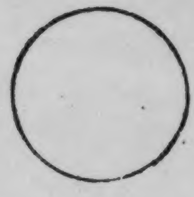


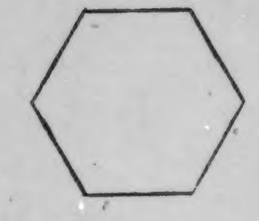
The following chart gives a suggested set of percents for the PLAYING cards.

METHOD

The deck of cards consists of 12 TRUMP cards--3 of each suit, and 40 PLAYING cards--10 of each suit. The symbols (suits) and equations should be placed on the cards as indicated below. A nylon tip marker works well for putting the suit design on the card. The problems can be typed or hand-written as desired.







1%	3%	5%	7%
22%	25%	34%	75%
100%	115%	200%	463%
.1%	62½%	5 3/8%	12 1/2%
½%	.75%	.003%	1/8%
2%	4%	6%	8%
23%	30%	38%	77%
110%	125%	250%	518%
42.3%	6¼%	4 3/4%	16 2/3%
¼%	7/8%	2.5%	1 1/8%

OBJECT OF THE GAME

First player to reach 21 points wins the game.

NUMBER OF PLAYERS

3, 4, or 5. However, if three players, then 1 card of each suit is to be taken from the 40 PLAYING cards.

RULES OF THE GAME

1. Shuffle the 40 PLAYING cards and 12 TRUMP cards separately.
2. Place TRUMP cards face down in center of the table.
3. Deal PLAYING cards, one at a time, to the players.
4. Each player sorts cards into like-suits.
5. To begin play, dealer turns up TRUMP card to determine suit.
6. Starting at dealer's left, each player plays card of that suit, calling out the decimal equivalent of the percent.

7. The player with the largest value card wins the trick.
8. Each player must play a card on every trick even when he cannot follow suit. However, he cannot win the trick if he cannot follow suit.
9. In case of a tie in the value of the card played, the winner of the trick is the first player to play one of the tieing cards.
10. Winner of the trick turns up a new TRUMP card to determine the suit and the game continues.

SCORING

Each player receives 1 point for each trick taken during the hand. Several hands may be played before the winner of the game is determined, unless the game is to be terminated before a goal of TWENTY-One is obtained. (This should be stated before the game is played.) Bonus points can be received by challenging or by predicting.

CHALLENGING

At any time during the game, an opponent can challenge the value of a card played by saying, "I challenge." Any other means of slowing the play of the game does not constitute a legal challenge (e.g. statements like "Hold it," "Wait a minute," or "Are you sure?" do not constitute a legal challenge).

a. If the wring value of a card is called, the challenger wins all tricks the challenged has accumulated that hand.
b. If the correct value of a card is called, the challenged player takes all of the tricks the challenger has accumulated that hand.
c. If an incorrect call is not challenged it can win the trick, if it has the largest value.

PREDICTING

a. After PLAYING cards are dealt and each player has arranged cards, each player predicts how many tricks he will take during that hand. If he predicts correctly, he receives 1 point for each trick plus an additional 5 points.
b. There is no penalty for a wrong prediction.
c. It is suggested that the predictions be recorded on paper.

ACTIVITY 3: Crossword puzzle (See Instruction Sheet III)

Suggested materials:
A copy of the puzzle for each member of the class.

Directions to student:
See Instruction Sheet III.

Suggested strategies:
Same as Activity 1 and 2. In addition point out that the decimal point and zeros need not be shown in the decimal equivalent of a percent such as 1500% (i.e. 1500% = 15.00 or 15)

Instruction Sheet III

Directions to student:

This crossword puzzle involves changing percents to decimals. A decimal point takes up a whole space by itself.

1	▨	2		▨	3
4	5		▨	▨	
6		▨	7	8	
▨	▨	▨	9		
10	▨	11	▨		▨
	▨	12	13	▨	14
15		▨	16		

Across

2. 30%
4. 460%
6. 8200%
7. 35%
9. 470%
12. 100%
15. 400%
16. 380%

Down

1. 48%
2. 60%
3. 15.7%
5. 20%
7. 40%
8. 390%
10. 4%
11. 10%
13. 30%
14. 80%

COMPETENCY: Write an equivalent fraction for a given fraction.

OBJECTIVES:

1. Given a fraction not in lowest terms, the student will be able to write the equivalent fraction in lowest terms.
2. Given a fraction the student will be able to write at least one equivalent fraction.

ACTIVITY 1:

Suggested materials:

Nine pie pans and nine different color disks to fit in the pans. Divide the pie pans into fractional parts. (halves, 3rds, 4ths, 5ths, 6ths, 8ths, 9ths, 10ths and 12ths) Label each 2/2, 3/3, 4/4, etc. Cut disks into corresponding fractional parts. (1/2's, 1/3's, 1/4's, etc.) Make enough sets so that every three or four students have a set. Directions to the student should be on a worksheet on which he may write.

Directions to the student:

Place the pie pans with the color wedges in front of you and examine them. Compare the sizes of different colored wedges.

1. How many 1/4 wedges does it take to make a 1/2 wedge?
   1.____________________
2. How many 1/8 wedges does it take to make 1/2 wedge?
   2.____________________
3. How many 1/6's does it take to make 1/2?
   3.____________________

We can write:
two "1/4's" as 2/4;
four "1/8's" as 4/8;
three "1/6's" as 3/6;
Each of these is equivalent to 1/2.
2/4 = 1/2
4/8 = 1/2
3/6 = 1/2

1/2, 2/4, 4/8, 3/6 are equivalent fractions. They are different names for the same fraction.
Look again at the pie pans and colored wedges.

4. How many "1/9's" make 1/3? 4.____________________

5. 1/3 = ____/9

6. How many "1/6's" does it take to make 1/3?
   6.____________________

7. ___/9 = 1/3 and ___/6 = 1/3

8. ___/9, ___/6, and 1/3 are equivalent fractions.

Use the pie pans and wedges to fill in the missing numerators or denominators.

9. ___/4 = 1/2

10. ___/12 = 1/2

11. ___/12 = 1/4

12. ___/8 = 1/4

13. ___/10 = 1/5

14. 2/___ = 1/5

15. 3/___ = 1/4

16. 2/___ = 1/3

If you have trouble with exercies 17 or 18, use the answers to problems 9 through 16 to help you.

17. Name two fractions that are equivalent to 1/4 =
__________ = __________

18. Name two fractions that are equivalent to 1/5 =
__________ = __________

Suggested strategies:

1. The student should be familiar with the parts of a fraction such as numerator and denominator.
2. The teacher may want to go over worksheets with the students.
3. The worksheets may be done in a small group.
4. Working with the teacher might be best if the students have not practiced working alone.
5. The teacher may supplement the worksheet with more problems on worksheets or from texts.

ACTIVITY 2:

Suggested materials:
Same as for Activity 1.

Directions to the student:
Using the pie pans and colored wedges please answer the following questions:

1. Take 2/5 (two 1/5 wedges put together). How many 1/10's does it take to make 2/5's?
   1. ____________________

2. How many 1/12's does it take to make 3/4's (three 1/4 wedges put together)?
   2. ____________________

Use the pie plates and colored wedges to fill in the missing numerators and denominators.

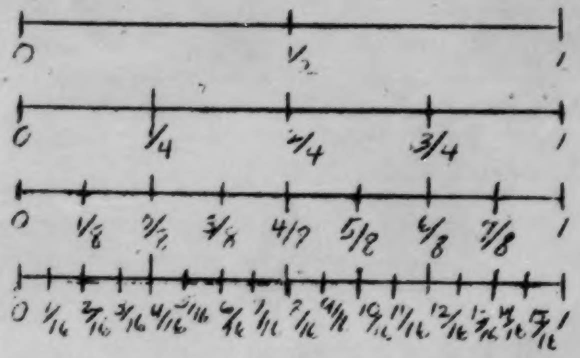
3. 5/6 = __/12
4. 2/3 = 4/__
5. 2/3 = __/12
6. 4/5 = __/10
7. 4/8 = 1/__
8. 4/6 = __/3
9. 3/4 = 6/__
10. 3/5 = 6/__
11. Name two equivalent fractions for 2/3 =
    ____________________ = ____________________
    Use exercises 1 through 10 to help you answer questions 11 and 12.
12. Name two equivalent fractions for 3/4 =
    ____________________ = ____________________

Suggested strategies:
If you did activity 1 with the students, you might try letting them do this activity in small groups of three or four.

ACTIVITY 3:

Suggested materials:
worksheet

Directions to the student:


0
1/2
1
0
1/4
2/4
3/4
1
0
1/8
2/8
3/8
4/8
5/8
6/8
7/8
1
0
1/16
2/16
3/16
4/16
5/16
6/16
7/16
8/16
9/16
10/16
11/16
12/16
13/16
14/16
15/16
1

Use the drawing above to help you decide which of the following fractions are equivalent. Place an equal sign (=) between each pair of equivalent fractions. (Do nothing if the pair is not equivalent.)

1. 1/4 ____ 2/8
2. 3/4 ____ 12/16
3. 14/16 ____ 7/8
4. 1/2 ____ 3/8
5. 7/8 ____ 10/16
6. 1/2 ____ 8/16
7. 5/8 ____ 10/16
8. 6/16 ____ 3/8
9. 4/16 ____ 2/8
10. 6/8 ____ 12/16
11. 6/8 ____ 3/4
12. 4/8 ____ 6/16

Circle the fractions that are in lowest terms.

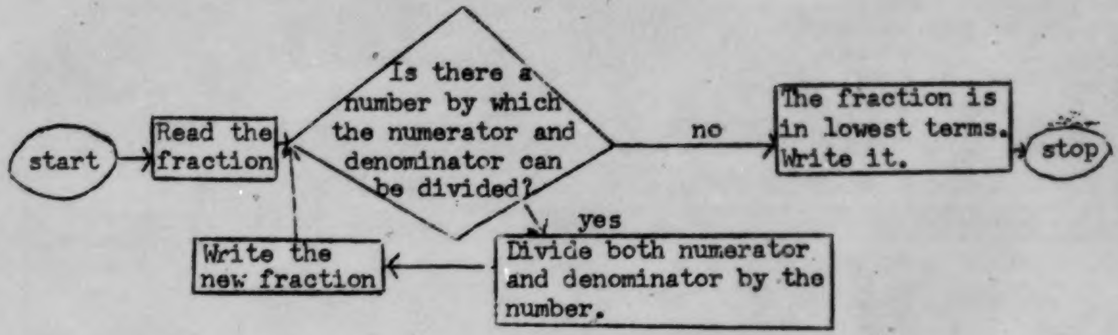
Suggested strategies:

1. The teacher should let the student become familiar with a ruler before doing this activity.
2. Let the student use the ruler to help answer the questions.

ACTIVITY 4:

Suggested materials:
worksheet

Directions to the student:


start
Read the fraction
Is there a number by which the numerator and denominator can be divided?
no
The fraction is in lowest terms. Write it.
stop
yes
Divide both numerator and denominator by the number.
Write the new fraction

Use the flow chart to help you complete the table.

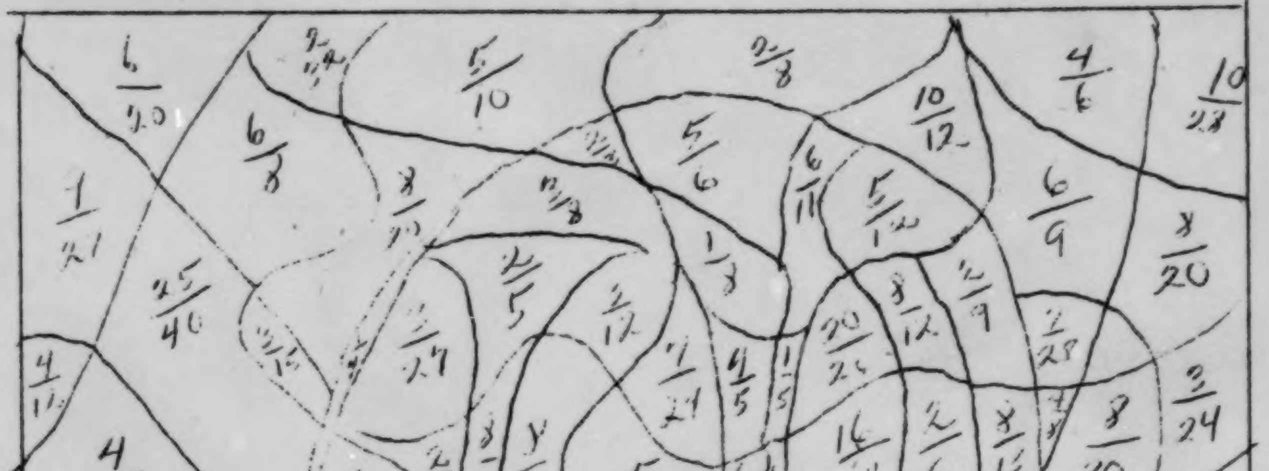
Fraction	Number by which both numerator and denominator must be divided.	Fraction in lowest terms.
6/8	2	3/4
4/8		
5/10		
12/16		
9/12		
6/9		
15/20		
8/10		
12/36		
10/20		
10/25		

Suggested strategies:
Students should be familiar with flow charting.

ACTIVITY 5:

Directions to Students:

Shade in the sections that have fractions in lowest terms.


6/20
5/10
2/8
4/6
10/28
6/8
5/6
10/12
1/27
8/10
6/11
5/12
6/9
8/20
25/40
2/5
1/8
2/9
4/12
2/27
2/12
4/5
8/12
2/28
3/24
4/26
11/33
2/4
8/30
5/15
16/32
2/6
8/12
8/30
2/15
2/5
7/10
2/10
9/64
4/7
7/14
7/11
9/64
4/15
3/4
7/10
2/5
2/3
5/3
2/3
8/16
5/9
4/7
11/13
4/9
2/10
1/2
2/1
4/9
6/13
11/13
4/5
2/21
2/4
3/21
8/12
2/5
2/3
20/26
2/4
2/6
5/12
12/48

ACTIVITY 6:

Suggested materials:

A set of 34 cards with one of the following fractions on each card.

10/20, 18/36, 4/12, 8/24, 4/6, 12/18, 4/8, 8/32, 9/12,

12/16, 5/25, 6/30, 10/25, 18/45, 6/15, 30/50, 16/20,

28/35, 3/18, 2/12, 25/30, 15/18, 2/16, 4/32, 9/24, 15/40,

15/24, 20/32, 31/27, 6/36, 10/18, 35/63, 35/45, and 14/18

One set for every five students.

Directions to the student:

Make a grid like the one below on your paper.

Choose any nine of the following fractions and put them in any order you wish, one in each of the nine boxes on your grid.

1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, 5/6,

1/8, 3/8, 5/8, 1/9, 5/9, 7/9.

After everyone has completed filling in his grid, one person turns over one of the cards. Find the equivalent fraction in lowest terms to the fraction on the card and then check your grid. If you have the fraction in lowest terms on your grid, cross it out. If not, do nothing to your grid. Turn over the next card and repeat the proceedure. The first one to get three in a row, across, down, or diagonally is the winner. When someone has three in a row everyone checks to be sure he is right. Make a new grid and play again.

Suggested strategies:

1. You can make this a class activity or play in small groups, say four or six students.
2. If the students are playing in groups, you may have to read the directions with them.
3. You can have several grids dittoed on one paper for the students to use.
4. Students may need help on how to fill in the grid. Show them an example.

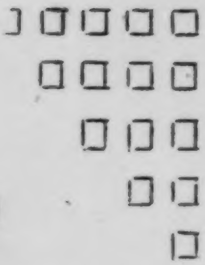
COMPETENCY: Use the standard algorithms for the operations of arithmetic of whole rational numbers.

OBJECTIVES:

1. After the student combines a series of objects representing the counting numbers, 1 through 9, he transfers this skill to abstract numbers and adds the single digit numbers vocally or by writing according to the directions given by the instructor.
2. The student will also write the problem horizontally as in the equation form, as well as vertically, and designate the ''addends'' and the ''sums.''

ACTIVITY 1: Combining groups to determine the total number of objects.

Suggested Material: Small squares of colorful plastic cut from scraps of Naugaheid (which may be procured from an upholsterer)



Directions to the Student: Represent the counting numbers 1 to 5 by placing some of the squares in rows of 1, 2, 3, 4, 5 squares, with the row of 5 squares on top and 1 square on the bottom. See figure at left.

Combine, or put together, the squares in the lowest 2 rows. How many squares are there? So you know that 1 + 2 = ______.

We call the numbers that were combined ''addends'' and the new number that you have is called the ''sum.'' So when we combine (or put together) we are adding.

Now add the next group to your new group. How many squares do you have? ______ So 3 + 3 = 6. What do we call this 6? ______ and the 3's are called ______.

The next group is 4. Add it to the sum you already have. Now how many squares do you have? ______ The total number of all the squares is ______.

We write the problem using the numbers: 1 + 2 + 3 + 4 + 5 = 15. Or we may write it vertically:

$$\begin{array}{r} 1\\ 2\\ 3\\ 4\\ +\,5\\ \hline 15 \end{array}$$

Suggested Strategies: After about 50 squares have been passed out to each student, pause 5 minutes to allow them to play with the squares on their desks. Then start the activity as found under ''Directions to the Student.''

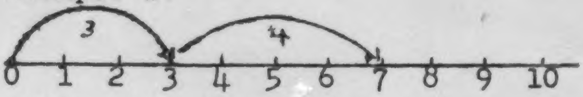
When the students have reached the sum of 15, if they are still interested, let them continue until all the digits have been added. If they are ready to finish adding the digits abstractly by using ohly the numerals, let them do so. Finish the lesson by having them indicate the addends and the sum in the last operation.

ACTIVITY 2: Addition by use of the Numberline.

Suggested Materials: A chalkboard model numberline or a transparency. A numberline slide rule may also be used.

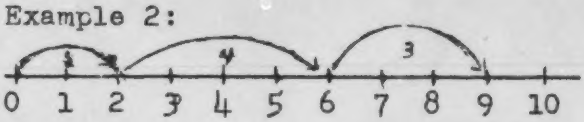
Direction to Student: Study the examples given here and referring to the numberline, work the following exercise on the accompanying numberline. Be sure to fill in the numbers and indicate the addends on the numberline; also complete the number sentence by filling in the sums.

Example 1:



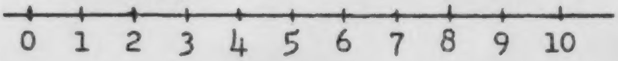
3 + 4 = 7

Example 2:



2 + 4 + 3 = 9

Use Example 1:

1) 

1) 3 + 2 =

2)

2) 1 + 4 =

3)

3) 5 + 3 =

4)

4) 6 + 2 =

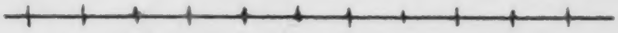
5)

5) 4 + 6 =

Use Example 2:

6)

6) 3 + 1 + 5 =

7) 

7) 2 + 3 + 4 =

8)

8) 4 + 5 + 1 =

9)

9) 6 + 0 + 3 =

10)

10) 5 + 2 + 2 =

Suggested Strategies: The attention of the student should be called to the fact that every unit must be just the same length as every other unit. If this activity is used in a class situation, a numberline drawn on a transparency can be thrown on the chalkboard. The students may write on the image produced; errors may be erased and the image will remain for further work.

The numberline slide-rule may be purchased ready for use. In a full class situation, the instructor and students can easily make their own from two strips of cardboard.

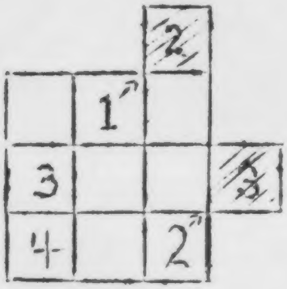
This is also an opportune time to give the students a review of the commutative property and the associative property if proper exercises are given them and the students may be asked to make up some problems of their own.

ACTIVITY 3: The Magic Square, an implementation to the addition of single digit numbers.

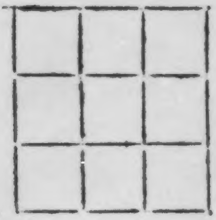
Suggested Materials: A prepared transparency grid, also pencil, ruler and paper for each student.

Direction to the Student:

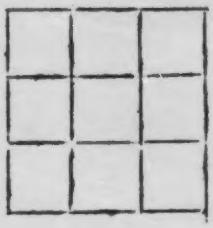
1. Draw a square with 9 cells and use the single digit numbers.
2. Place the number 1 in the middle cell of the top row.
3. By moving diagonally up to the right begin to fill in the other spaces, but this puts the number out of the square. When this occurs, simply drop to the last cell in that column or row.
4. Move diagonally from 3 up to the right, but that cell is already occupied. When this occurs go back to the starting point (in this case 3) and put the number 4 in the cell below the 3. This rule also applies if you move out of the square on a diagonal.
5. Continue until all cells are filled. Now check for accuracy by adding each row, each column and each diagonal. Is the sum the same in each case ______?



6. Repeat in Figure 2



7. Repeat, but begin with the number 2 and continue through the number 10.



Suggested Strategies: The center cell will be 1/2 the sum of the adjoining cells in the row, column, or diagonal. These same rules will hold for all squares having an odd number of cells. Consecutive odd numbers, even numbers, or multiples of a number may be used to fill the cells.

COMPETENCY: Addition and subtraction facts

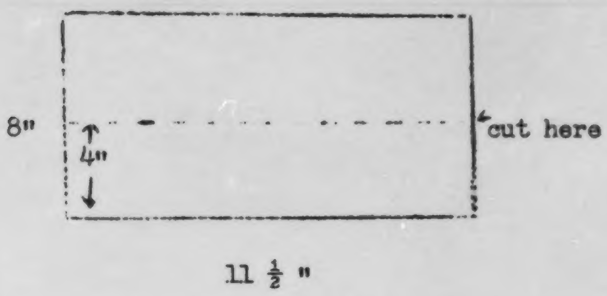
OBJECTIVES: The student will be able to state addition facts of numberals 1-10.
The student will be able to state subtraction facts of one digit numerals from numerals less than 20.

ACTIVITY 1: Build an addition slide rule.

Suggested materials:
Construction paper size 8 x 11 ½
or manila folder, pen or pencil, 12 inch ruler.

Directions to the student:

1. Cut a strip of paper from the construction Paper or from the manila folder 4 inches wide so you have a piece 4 inches by 11 ½ inches.



2. Cut a second strip 2 inches wide.

3. On both pieces of paper, mark off units of ½ inch. Draw a line lengthwise, one inch from the top on both pieces. See diagram 1.

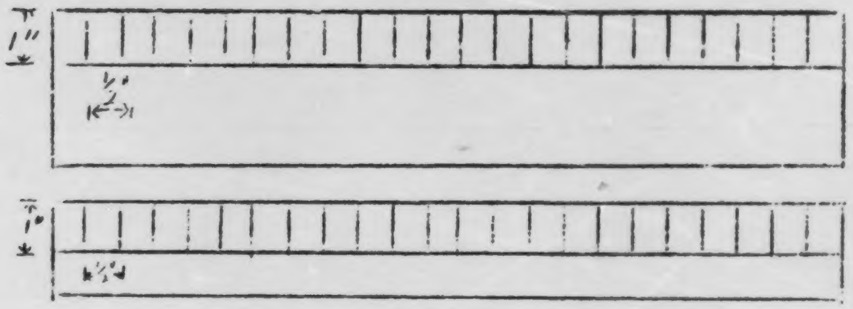


Diagram 1

4. Leave the first space blank on both pieces. Draw a red dot in the second space of the 2 inch strip and draw a black dot in the second space of the 4 inch strip. Starting in the third space on both strips, write the numerals 1-20 in the successive spaces. See diagram 2.

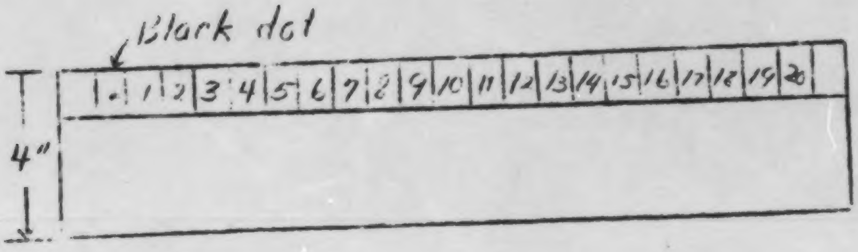

Black dot
4"
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Diagram 2

5. Fold the 4 inch strip along the line drawn 1 inch from the top so that the numerals will be in the front. See diagram 3.

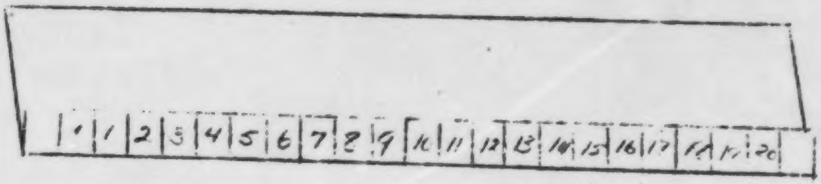

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Diagram 3

6. We will call this the "rule" of our slide rule. Place the 2 inch strip on the fold of the 4 inch card. This is the "slide" or your slide rule. Your completed slide rule will be as in diagram 4.

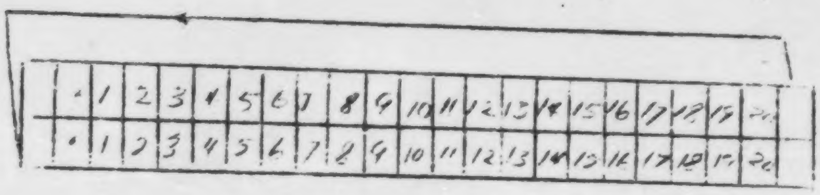

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Diagram 4

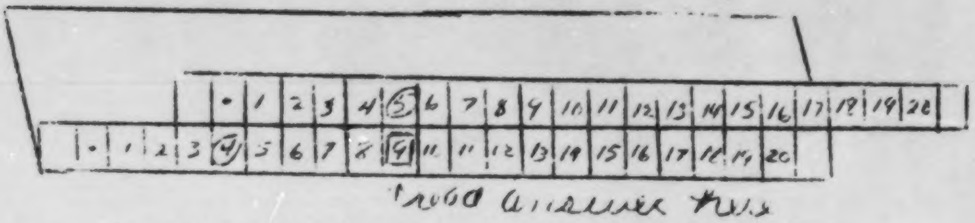
ACTIVITY 2: Adding one digit numerals

Suggested materials:
The addition slide rule constructed in Activity 1.

Directions to the student:
Assume we want to add 4 and 5.

1. Place the red dot of the "slide" above the numeral 4 on the "rule."
2. Read the numeral on the "rule" below the 5 on the "slide." This numeral is 9, thus 4 + 5 = 9. See diagram 5.

Diagram 5


• 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
• 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
read answer here

3. Find the following sums using your slide rule.
   (a) 7 + 5 (c) 9 + 8
   (b) 6 + 2 (d) 3 + 3
4. Make up some problems of your own.

Suggested strategies:
The teacher may use the slide rule for oral practice or may have written expercises on ditto paper for the student to complete.
The teacher should stress the importance of aligning the blocks on the "slide" with the blocks on the "rule."

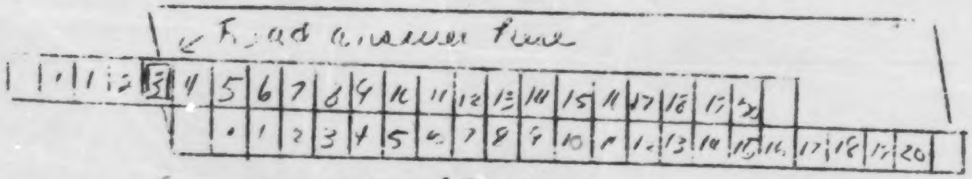
ACTIVITY 3: Subtracting one digit numerals from numerals less than 20.

Suggested materials:
The addition slide rule constructed in Activity 1.

Directions to the student:

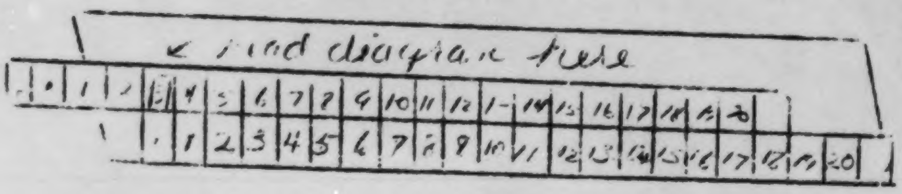
1. Assume we want to subtract 8 from 13.
   a. Place the 13 on the slide above the 8 on the rule.
   b. Read the answer 5 on the "slide" above the black dot on the "rule." Thus 13 - 8 = 5. See diagram 6.

Diagram 6


Read answer here
• 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
• 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

2. Subtract 6 from 9
   a. Place the 9 on the "slide" above the 6 on the "rule."
   b. Read the answer 3 on the "slide" above the black dot on the "rule." Thus 9 - 6 = 3. See diagram 7.

Diagram 7


← read diagram here
• 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
• 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

3. Use your slide rule to find the difference of the following:
   (a) 18 - 9 (c) 15 - 6
   (b) 14 - 8 (d) 13 - 2

4. Make up some problems of your own.

Suggested strategies:

The teacher may use the slide rule for oral practice or may have written exercises on ditto paper for the student to complete.

COMPETENCY: Use the standard algorithms for the operations of arithmetic of whole rational numbers

OBJECTIVES:

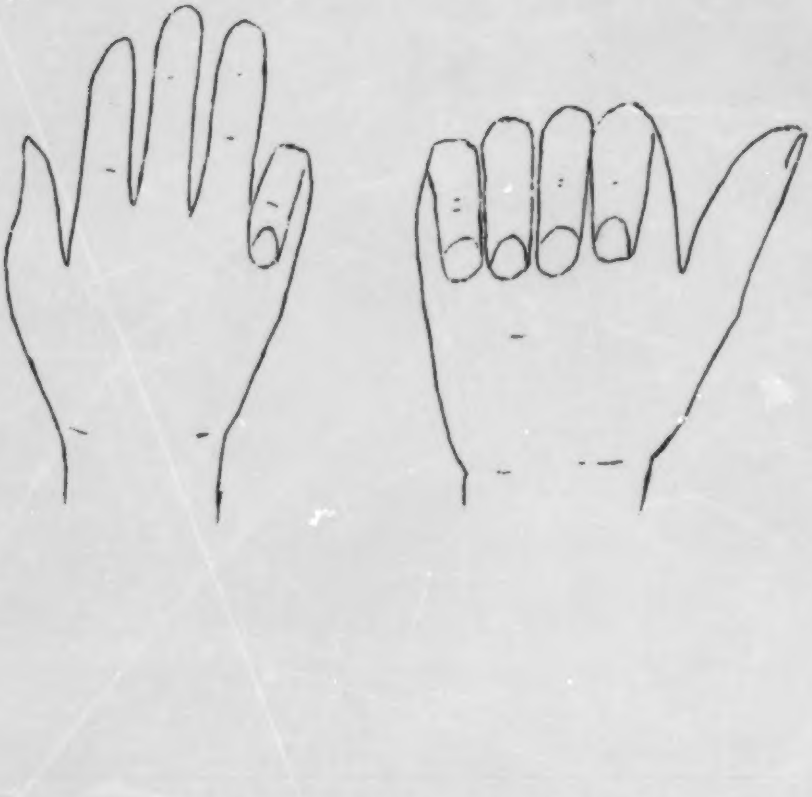
The student will find the product of any two numbers between 5 and 10, given (knowing) the multiplication facts up to 5 x 5 and the addition facts.

ACTIVITY 1:

Suggested materials:
None

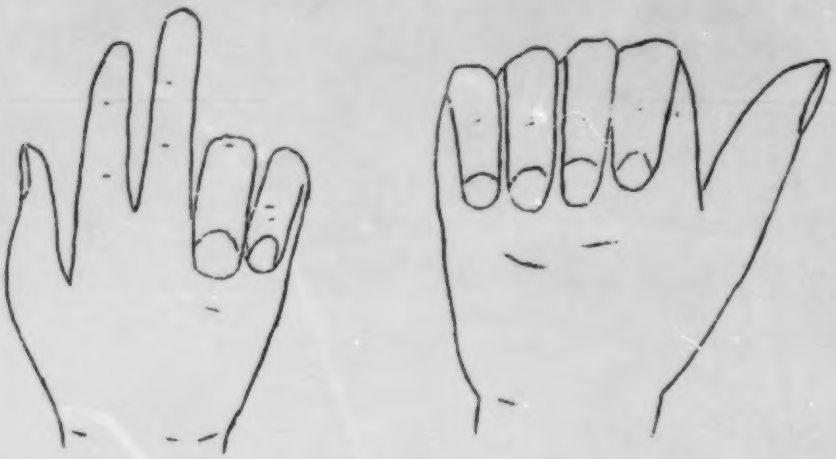
Directions to student:
Here is a way some people in Europe multiply two numbers that are between 5 and 10.
Pick two numbers between 5 and 10, say 6 and 9.
6 x 9 is how many?
To find the answer, think 6 is 1 more than 5. Bend 1 finger down on your left hand.
Nine is 4 more than 5. Bend 4 fingers down on your right hand.

Your hands should now look like those below:



The number of fingers bent down on both hands gives the tens.
So the answer is 5 tens and some ones.
Multiply the unbent fingers on each hand for the ones. This gives as 4 x 1 = 4 ones.
The answer is 5 tens and 4 ones 54.

Let's try another example.
7 x 9 = ?
7 = 5 + 2, so bend 2 fingers down on your left hand.
9 = 5 + 4, so bend 4 fingers down on your right hand.
Your hands now look like this



6 fingers are down. This gives us 6 tens.
3 fingers are up on one hand and one on the other. This gives us 3 x 1 = 3 ones.
The answer is 6 tens and 3 ones or 63.
Therefore 7 x 9 = 63.
Remember:

1. Bend fingers down for extra numbers over 5.
2. Add the number of bent fingers to get tens answer.
3. Multiply the number of unbent fingers to get the ones answer.

Now try:

6 x 7   8 x 9   7 x 7   9 x 9   7 x 9

8 x 7

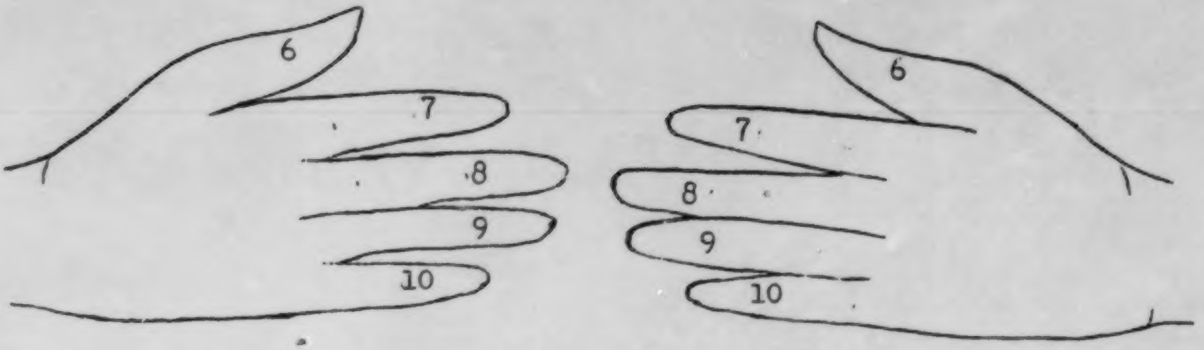
Suggested strategies:

This activity can be extended to include the 5's, but the addition becomes a little more cumbersome.
For example: 5 x 7 has 0 fingers down on the left hand, 2 fingers down on the right hand. This gives us two tens plus 5 x 3 = 15 ones. Two tens and 15 ones is 35 which is 5 x 7.
The same is true for 6 x 7.

ACTIVITY 2:

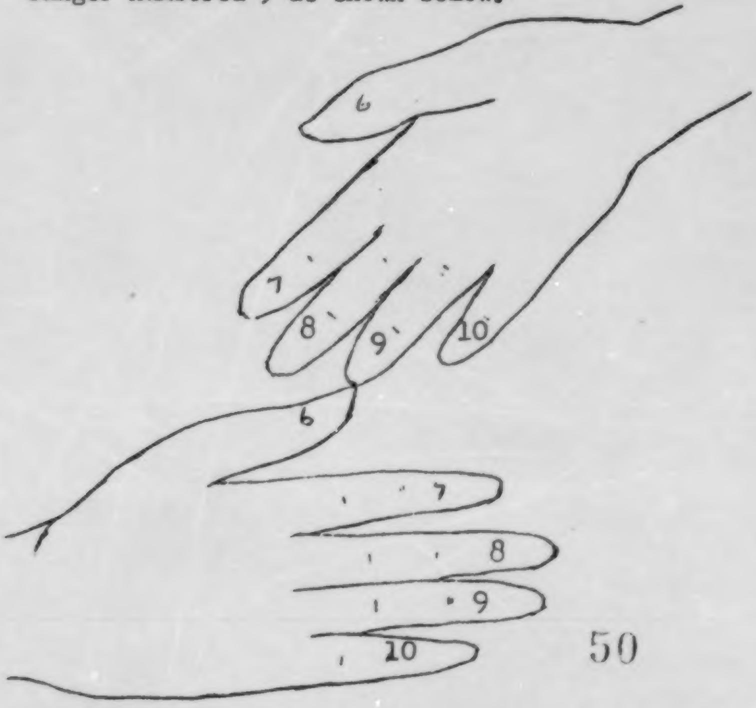
Suggested materials:
None

Directions to student:
Here is a way to multiply two numbers between 6 and 10. Number the fingers on each hand, 6, 7, 8, 9, 10 as shown below.

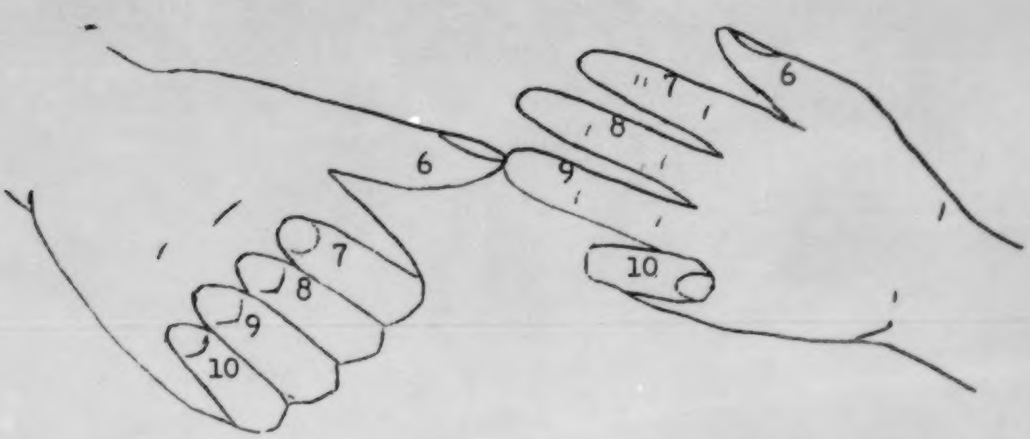


6 x 9 = ?

To find the answer, place the finger numbered 6 against the finger numbered 9 as shown below.



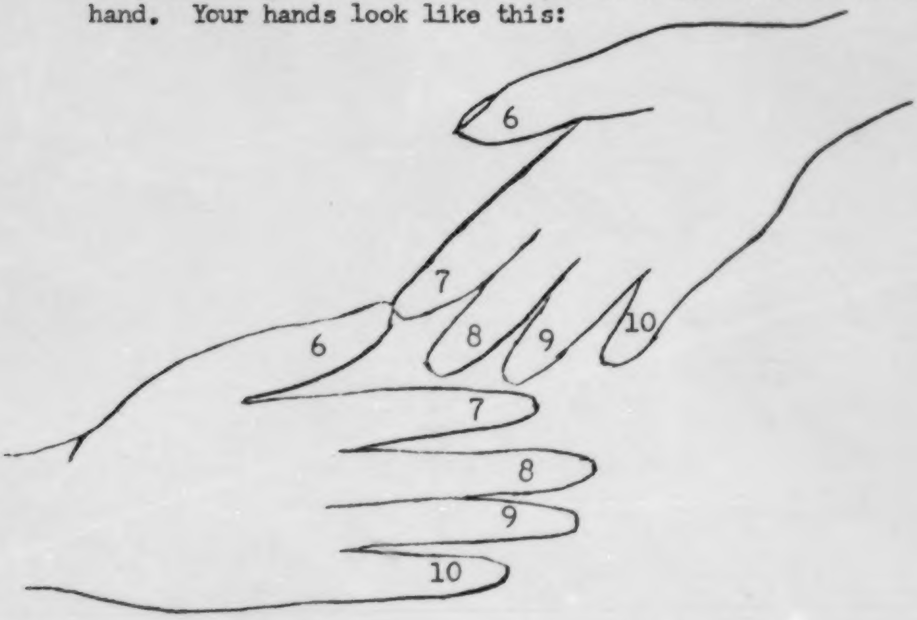
Close all fingers below the 6 and 9. Your hands now look like this:



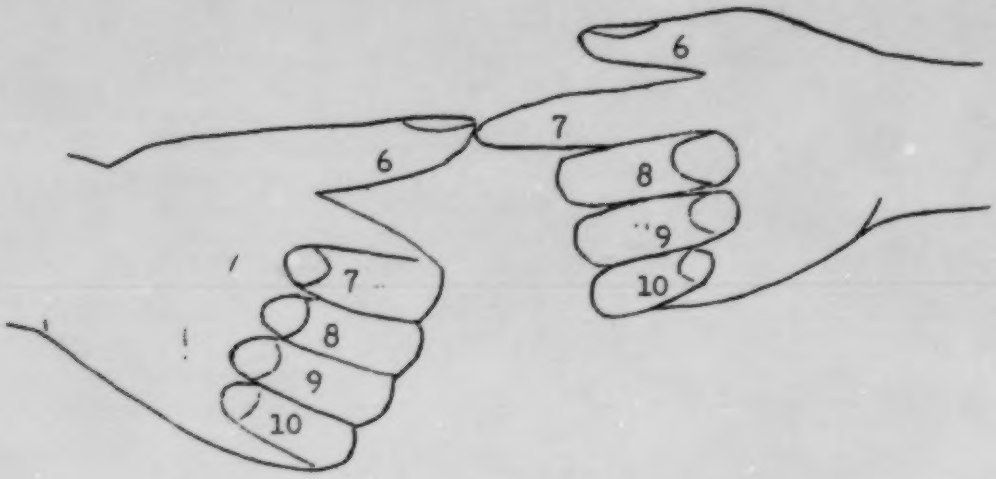
The total number of open fingers gives us the number tens in our answer. There are 5 tens.

Multiply the number of bent fingers in one hand times the number of bent fingers in the other. This gives us the number of ones in the answer. 4 x 1 = 4 ones
The answer has 5 tens and 4 ones or is 54.
So 9 x 6 = 54

Find 6 x 7.
We touch finger 6 on one hand against finger 7 on the other hand. Your hands look like this:



Close all fingers below 6 and 7. Your hands now look like this.



There are three open fingers so there are 3 tens in our answer.
There are 3 fingers closed on one hand and 4 on the other.
3 x 4 = 12 which gives us 12 ones or 1 ten and 2 ones.
3 tens and 1 ten and 2 ones gives us
4 tens and 2 ones or 42.

So, 6 x 7 = 42

Now try:

7 x 7	6 x 8	7 x 9	9 x 9	8 x 8
7 x 8				

Suggested strategies:
Most of the problems will work like the first example.
Example 2 is the only one that will give you ten or more ones.
That is why it is shown.

COMPETENCY: Operation and Properties

Use the standard algorithms for the operation of arithmetic of whole rational numbers.

OBJECTIVES:

The activities in this packet satisfy the following behavioral objectives:

1. Given a simple multiplication problem, the student will be able to give an inverse relationship and find the answer.
2. Given a simple division problem, the student will be able to give an inverse relationship and find the answer.
3. Given a simple division problem in which the divisor and dividend are only one number, the student can find the quotient.
4. Given a division problem in one of three forms, the student can recognize which member of the problem is the divisor, and which is the dividend. He can solve the problem.
5. Given a division problem in which the divisor is a one digit number and which has no remainder, the student can find the quotient.
6. Given a division problem in which the divisor is a one digit number and which has a remainder, the student can find the answer. He can write the quotient quotient showing the remainder as a fraction.
7. Given a division problem with a two digit divisor-- units digit less than 5, the student can find an answer.
8. Given a division problem with a two digit divisor-- units digit equals 5, the student will find an answer.
9. Given a division problem with a two digit divisor-- units digit greater than 5, the student will find an answer.

Suggested strategies for all of the Operation and Properties Units:

Each level of development in this unit has a behavioral objective.

Worksheets prepared have no more than ten problems. The problems have been selected so that the columns are of comparable difficulty.

Explanation of the Worksheets in the Unit:

The worksheets for this unit with the exception of Activity 1 are ten problems long. These ten problems have been arranged from the simple to the more difficult. Problem number 10 on some sheets can be a review problem or a problem introducing new material.

The problems are arranged on the sheet in such a way that the odd numbered problems are comparable with the corresponding (across the page) even number. This is done so the teacher may use only half the sheet at a time.

The worksheets are made up of three types of problems.

1. Those with remainder (0)--"even problems"
2. Those with remainders
3. Composite page --made up of both types 1 and 2.

These various types will be designated on the teacher strategy pages dealing with each activity.

Bonus pages are also included for use at the discretion of the teacher.

Teacher strategies:

There are several ways that the worksheets in this unit could be used in the classroom.

Type 1:

1. A concept could be introduced by the teacher.
2. The teacher then works several problems using the concept, calling on the student when possible for responses.
3. The students are given a problem or several problems to work. The teacher then acts as a monitor watching how the student works, helping when necessary, encouraging if needed or re-explaining the idea being used, working through the problems with the student.
4. The correct solution can be placed on the board or overhead. Ask the students to compare their work with the correct solution.
   The following questions could be asked.
   a. Does your problem look like the correct solution? (If it does, offer a word of praise as these students usually need reinforcement of the feeling one gets from being correct.)
   b. If your problem does not look like the correct solution. Where does it look different?
   c. Ask those students who do not have the correct answer to raise their hands and the teacher should work through the incorrect problems with the student.

d. When all have a sufficient understanding and seem confident, give the students a worksheet. Have them fold it in half or present it already cut in half. Instruct them to work these five problems and check their answers.

e. When all have finished have them hand their worksheet to a fellow student. Each student will now check the work of another student. If they find errors in the work, they must explain to the other student where he made his error. The grader should explain how to correct the problem. Verbalizing seems to help their understanding. This often can become an argument time and the teacher will have to be alert to this and step in when necessary.

f. The correct solutions are made available to the student and they can then compare their checked work with the correct solution. The teacher could answer any remaining questions.

g. Give the other five problems and these can then be graded or handled as the teacher desires.

Type II:

The worksheets could be handled as a timed drill. Giving five of the problems first as practice drill and then the remaining five as the "real" thing.

Type III:

They could be handled as a review expercise for those who already feel competant and do not need the intensive practice.

OBJECTIVES:

1. Given a simple multiplication problem the student will be able to give an inverse relationship and find the answer.
2. Given a simple division problem, the student will be able to give an inverse relationship and find the answer.

ACTIVITY 1:

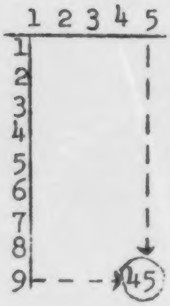
Suggested materials:

worksheet for multiplication grid.

Suggested strategies:

This activity has been planned as an activity for the entire class. It is possible that the activity is too easy for the class, if it is it could be omitted. Have the students fill in the multiplication grid explaining that the product in any empty box is the result of multiplying together the number which leads the column and the number which heads the row in which the box is located. It is suggested that a transparency could be made of the multiplication grid and the discussion of these problems could be accomplished by the use of this grid on the overhead.

When the table has been completed the following conversation could take place between the teacher and students or student.



Teacher: "If I ask you the question, What is 45 ÷ 5 ? (This should be put on the board) We could find the answer in our chart. Locate a 5 in the table. It can head a row or lead a column. If you have located the 5 which leads a column, run your finger down that column until you locate 45. Now to find the answer to our original problem 45 ÷ 5, you shoud move your finger to the left in the row and locate the number which heads that column."

The above procedure could be carried out several times until the students are familiar with it. The teacher could then continue by asking several students in the class other simple problems. When all seem to understand how the chart works the teacher might say,

Teahcer: "Remember, when we filled in this chart we were using what operation?"

Hopefully the response from the students will be multiplication. Then the teacher should continue--

"What operation are we using now?"

"division."

The teacher continues:

"There seems to be a relationship between multiplication and division. Can anyone tell me what they see?"

Responses may vary but typical responses will be:

"One seems to 'undo' the other."
"Division is part of a multiplication problem."

The teacher then could conclude the discussion by presenting the idea that division is the inverse operation of multiplication. Also she could present the student with the idea that by effectively using this chart of multiplication facts, a student can divide. What the teacher is trying to get the student to see is that division (with zero remainder) is the renaming of a product and one factor in terms of the missing factor.

As much practice as the teacher thinks is necessary should be given. It is suggested that the teacher works with chart and student until they seem proficient. She should stop before boredom sets in.

This activity could also be used on an individual basis with students that are unsure of their facts, or as a review.

X	0	1	2	3	4	5	6	7	8	9
0										
1		1								
2			4							
3				9						
4					16					
5						25				
6							36			
7								49		
8									64	
9										81

Instructions: Multiply to fill in the boxes.
Example: 2 x 2 = 4, 2 in row x 2 in column
3 x 3 = 9, 3 in row x 3 in column
4 x 4 = 16, etc.

ACTIVITY 2:

Suggested materials:

Colored rods of different sizes. These can be prepared by the teacher by using a ditto master and colored paper.

Suggested sizes: 12 inch rod
10 inch rod
8 inch rod
9 inch rod
2 inch, 4 inch, 5 inch, 3 inch,
1 inch, 6 inch

Directions to the student:

1. Select a 12 rod.
   Question: How many 4 rods can you use to completely cover the 12 rod?____________________
   Write a problem which shows this relationship.
   Example: Possible student responses:
   $3 \times 4 = 12$ or
   $12 \div 4 = 3$

2. Select a 10 rod.
   Question: How many 5 rods do you use to cover the 10 rod?____________________
   Write a problem which shows what you have done.

Suggested strategies:

The above procedure could be continued by the teacher for as long a time as it is felt the students need to comprehend the concept--that division is the renaming of a product and 1 factor (multiplier) in terms of the missing factor.

This activity could be conducted with just part of the class as a remedial exercise for those who cannot get the concept from the original discussion or it could be used as an introduction to this section.

The rods can be used very effectively on the overhead. The manipulation of them shows up quite clearly and it can be an effective method of presentation to the whole class.

A worksheet for this type of activity could be constructed and students could work on this at their seats using this as a math lab experiment.

ACTIVITY 3:

Suggested materials:

Colored rods as indicated in Activity 2.
Whole number lines 0 - 20

Directions to the student:

On the number line place a 12 rod so one end touches 0. Below the number line place as many 4 rods as are necessary so the 4 rods are as long as the 12 rod.

How many 4 rods have you used?________________________

What number on the number line is at the end of the 12 rod?________________________

What number on the number line is at the end of the 4 rod?________________________

How many four rods did it take to make a rod as long as the 12 rod?________________________

This problem can be stated two ways.

1. Three 4 rods = one 12 rod
   or 3 x 4 = 12
   This is a ________________________ problem.

Suggested strategies:

Have the students make five problems of their own using the rods. Have them write the problems which they have made using the rods as:

1. multiplication problems, and
2. division problems.

ACTIVITY 4:

Suggested materials:
Centimeter grid paper or grid paper the teacher or student may make or have in the classroom.

Directions to the student:
Place numbers on the grids from 1 thru 50.
Cut the grids and paste or tape the strips on paper together in such a way that it looks like this. You should have the numbers from 1 thru 50 in a straight line.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Using the strip of paper you may solve division problems this way.

15 ÷ 5 = ________

Take your strip of paper.
Look at the numbers 1 thru 15.
Can you fold these 15 numbers into strips with 5 numbers in each strip?
How many equal strips are there?________________________
Did you find 3 strips of 5 squares each?
You have just found the answer to the problem 15 ÷ 5 = 3

Here are some problems for you to solve using your long strip.

1. 16 ÷ 4 = ________
2. 25 ÷ 5 = ________
3. 50 ÷ 10= ________
4. 18 ÷ 6 = ________
5. 49 ÷ 7 = ________
6. 36 ÷ 9 = ________
7. 35 ÷ 5 = ________
8. 40 ÷ 8 = ________
9. 12 ÷ 3 = ________
10. 50 ÷ 25= ________

OBJECTIVES:

1. Given a simple division problem in which the divisor and quotient are only one digit, the student can find the quotient.

ACTIVITY 1:

Suggested materials:

worksheet containing 50 simple division problems.

Suggested strategies:

The teacher administers drill of basic division facts. Time limit 10 - 15 minutes dependent upon the teacher's understanding of her class. Inform the student that he may use his grid if he needs it. Since this type of student often lacks self confidence, a simple tool such as this can help them gain confidence. Most will find the grid is not necessary.

When the drill has been completed, grade so the students may see how they have done. For those who the teacher feels need further work on these simple problems, she should review the work on the grid and then give these students the second worksheet of this activity. A second worksheet could be used as a review sheet if needed.

The student could use any of the manipulative tools described in the prior unit to find the answers. If this is done, it may be necessary to remove the time limit on the sheet.

Activity 1

worksheet A

Divide:

24 ÷ 4 =
42 ÷ 6 =
9 ÷ 3 =
7 ÷ 7 =
63 ÷ 7 =
10 ÷ 2 =
72 ÷ 7 =
49 ÷ 7 =
3 ÷ 3 =
72 ÷ 8 =

30 ÷ 6 =
16 ÷ 2 =
56 ÷ 8 =
6 ÷ 1 =
48 ÷ 8 =
5 ÷ 1 =
20 ÷ 5 =
14 ÷ 7 =
20 ÷ 4 =
54 ÷ 6 =

24 ÷ 3 =
81 ÷ 9 =
12 ÷ 4 =
2 ÷ 2 =
10 ÷ 5 =
64 ÷ 8 =
12 ÷ 3 =
25 ÷ 5 =
9 ÷ 9 =
7 ÷ 7 =

3 ÷ 1 =
30 ÷ 5 =
36 ÷ 9 =
72 ÷ 8 =
54 ÷ 9 =
3 ÷ 3 =
42 ÷ 7 =
10 ÷ 2 =
63 ÷ 7 =
28 ÷ 4 =

28 ÷ 7 =
8 ÷ 2 =
36 ÷ 4 =
15 ÷ 5 =
24 ÷ 6 =
36 ÷ 9 =
36 ÷ 4 =
54 ÷ 9 =
18 ÷ 3 =
18 ÷ 2 =

Activity 1

Worksheet B

Divide

18 ÷ 2 =
21 ÷ 7 =
6 ÷ 3 =
35 ÷ 9 =
6 ÷ 2 =
36 ÷ 9 =
12 ÷ 6 =
9 ÷ 9 =
30 ÷ 6 =
6 ÷ 1 =

16 ÷ 8 =
32 ÷ 4 =
40 ÷ 8 =
3 ÷ 3 =
45 ÷ 5 =
4 ÷ 4 =
10 ÷ 5 =
63 ÷ 9 =
24 ÷ 3 =
54 ÷ 6 =

15 ÷ 3 =
72 ÷ 8 =
14 ÷ 2 =
24 ÷ 6 =
18 ÷ 6 =
24 ÷ 8 =
4 ÷ 2 =
27 ÷ 9 =
30 ÷ 6 =
25 ÷ 5 =

7 ÷ 1 =
54 ÷ 9 =
2 ÷ 1 =
4 ÷ 1 =
45 ÷ 9 =
36 ÷ 4 =
1 ÷ 1 =
42 ÷ 6 =
24 ÷ 4 =
12 ÷ 3 =

35 ÷ 5 =
48 ÷ 6 =
40 ÷ 5 =
28 ÷ 4 =
8 ÷ 4 =
72 ÷ 9 =
32 ÷ 8 =
9 ÷ 3 =
28 ÷ 7 =
64 ÷ 8 =

OBJECTIVES:

1. Given a division problem in one of three forms, the student can recognize which member of the problem is the divisor, and which is the dividend. He can solve the problem.

Suggested materials:

Worksheet
Dictionaries
"D" encyclopedias
Simple Math Books

Directions to the student:

On the worksheet.

Suggested strategies for the unit:

The teacher should have in the room, dictionaries, encyclopedias (d), or other Math Books. On the board or on a transparency place these words:

Divisor
Dividend
Quotient

Have the students using the means available in the room find and write a definition or give an example that explains these terms. Have them locate these terms in a division problem. The teacher could then have the students present their discoveries to the class, by putting them on the board or by using the overhead.

If the student has located these terms in a problem using this symbol have them locate the terms in a problem of form ÷ and a fraction.

The worksheet accompanying this activity has some problems that they can illustrate their knowledge.

When they have these facts in mind refer back to the grid and have the students remember how they found their answers then lead them to see that the dividend is actually a product thus the check for a division problem is a multiplication problem using the divisor x the quotient.

Inform the students that all division problems should be checked. This procedure acts as a reinforcement of multiplication and also helps their self-confidence since they can tell whether their answer is correct or not. If check doesn't give the correct answer, return to the division problem, recopy it and try again.

Activity 1

Worksheet C

Write a definition of:

1. Divisor

2. Dividend

3. Quotient

In the following examples name the divisor, the dividend and the quotient.
Check each problem.
Check

1. $4 \div 2 = 2$ _____ = divisor ____________________
_____ = dividend
_____ = quotient

2. $\frac{8}{2} = 4$ _____ = divisor ____________________
_____ = dividend
_____ = quotient

3. $6 \div 3 = 2$ _____ = divisor ____________________
_____ = dividend
_____ = quotient

4. $18 \div 3 = 6$ 6 = _______ ____________________
3 = _______
18 = _______

5. $\frac{12}{3} = 4$ 3 = _______ ____________________
4 = _______
12 = _______

Worksheet C

Check

6. 25 ÷ 5 = 5 5 = ______ ______________

25 = ______

5 = ______

Write and solve one division problem using these signs:

7. $\overline{)\quad\quad}$

8. ______

9 ÷

Check your problems:

7. 8. 9.

OBJECTIVES:

1. Given a division problem in which the divisor is a one digit number and which has no remainder, the student can find the quotient.

ACTIVITY 1:

Suggested materials:
Examples of division problems
Worksheets for students

Suggested strategies:
Introduction to Activities 2, 3, and 4.

The teacher should inform the student that the problems in this activity have remainders of zero or as the student may say--"Come out even."
The teacher should also remind the student that each of the problems in the accompanying worksheet require a check. Work these examples or similiar examples with the students. Some of the students may know who to do these problems. Let them have an opportunity to show how to do these problems. They could do them on the board or at the overhead. Some may not even need these expercies and should be given some more difficult problems.

Example problems:

1. $$\begin{array}{r} 19 \\ 4\overline{)76} \\ \underline{4} \\ 36 \\ \underline{36} \\ 0 \end{array} \qquad \begin{array}{r} 19 \\ \underline{\times 4} \\ 36 \end{array}$$

2. $$\begin{array}{r} 12 \\ 8\overline{)96} \\ \underline{8} \\ 16 \\ \underline{16} \\ 0 \end{array} \qquad \begin{array}{r} 12 \\ \underline{\times 8} \\ 96 \end{array}$$

3. $$\begin{array}{r} 13 \\ 9\overline{)117} \\ \underline{9} \\ 27 \\ \underline{27} \\ 0 \end{array} \qquad \begin{array}{r} 13 \\ \underline{\times 9} \\ 117 \end{array}$$

The teacher can point out in the examples that division follows a pattern. i.e.

See example 1. 4 x _____ = some number = to 7 or smaller than 7.

4 x 1 = 4

4 x 2 = 8 which is too large, hence the number 1 is selected as the other multiplier.

2. The 1 is placed above the 7. It is possible to explain the placing of 1 above the seven by reminding the student of the procedure used with the grid. The student should be encouraged to think through the multiplication problem 1 x 4 = 4 and places the 4 beneath the 7.

3. The problem is now one of subtraction of 7 - 4 = 3.

4. Now the student must deal with a new division problem. 36 ÷ 4 = ______ or 4 x ______ = 36 or ______ = 9. He may refer to this as "bringing down the six," to make the number. If he chooses to stop at this point remind him that we are looking for two numbers whose product is 76. IF he stops here before bringing down the six he has only the produce 4.

5. He then subtracts 36 or 9 x 4 from 36 and his problem is finished.

6. Encourage him at this point to look at his problem. Discuss with him how he knows when to stop dividing. This will be discussed more later.

He can be encouraged to think of the pattern of division--that of multiplication then subtraction until as in these specific cases, the subtraction result is zero or as he may say, "It came out even."

Additional work:

Suggested materials:
Worksheet

Directions to the student:
Introduction to Activities 2, 3, and 4.

Suggested strategies:

The following exercises could be used as bonus material or included as part of this activity. Some divisibility tests are more obvious than others. It is suggested that a teacher determine on the basis of her students the tests or test which she feels her students can handle. These tests have been classified according to student ability.

Level I------Divisibility tests for 2 and 5.
Level II-----Divisibility tests for 3 and 9.
Level III----Divisibility tests for 4, 6, and 8.

The rules for these may be stated but the student should not be relied on to remember it as stated. Examples for each rule should be given and then the student should work through these with the teacher before working alone.

Activity 4

Worksheet A

Divide and check:

$8\overline{)96}$

$9\overline{)117}$

$6\overline{)384}$

$3\overline{)288}$

$4\overline{)352}$

$2\overline{)66}$

$5\overline{)125}$

$7\overline{)392}$

$1\overline{)161}$

$10\overline{)100}$

Activity 4

Worksheet B

Divide and Check:

$2\overline{)84}$

$5\overline{)810}$

$7\overline{)9247}$

$8\overline{)872}$

$4\overline{)872}$

$3\overline{)75693}$

$9\overline{)9045}$

$6\overline{)9468}$

$1\overline{)41613}$

$10\overline{)80120}$

ACTIVITY 2:

Suggested materials:
worksheet

Suggested strategies:
Level 1 A. Divisibility Test for 2.

A number is divisible by 2 only if it ends in 0, 2, 4, 6, or 8. The teacher can use the above and lead the student to the idea or remind him that 0, 2, 4, 6, or 8 are even numbers and thus help him state this rule simply as: All even numbers can be divided by 2 evenly.

Level 1 B. Divisibility test for 5.

A number is divisible by 5 only if it ends in 5 or 0. Here the teacher can use money as a way of presenting the topic or he can choose to take the basic multiplication facts for 5 and lead the students to the fact that the products all end in 0 or 5 and thus the rule.

Additional material--Divisibility Test (2,5)

Level I

1. Can 2 divide 8352?
2. Can 2 divide 58796?
3. Can 2 divide 675?
4. Can 5 divide 675?
5. Can 5 divide 976,000?
6. Draw a circle around the following numbers which can be divided by 2.

742 73 28,465 764,750

7. Draw a box around the following numbers which can be divided by 5.

687 9050 5,275 28,206

ACTIVITY 3:

Suggested materials:
worksheet

Suggested strategies:
Level 2 A. Divisibility Test for 3.

A number can be divided evenly by 3 if the sum of its digits is divisible by 3. In presenting this rule it is feasible for the teacher to return to the basic multiplication facts and show the above is true. State the rule after enough examples have been worked so that the student understands the concept before presented with the rule.

Level 2 B. Divisibility Test for 9.

A number can be divided by 9 only if the sum of the digits can be divided by 9.
At this point perhaps the student has already discussed this particular characteristic of 9 when he was multiplying. It would be very easy to then proceed to the rule. If he has not, then the teacher again should help him see the concept before the words or rule are before him.

Additional material--Divisibility Test for 3, 9.

Level II

1. Can 3 divide 83,592?
2. Can 3 divide 468?
3. Can 9 divide 468?
4. Can 9 divide 83,592?
5. Can 9 divide 117?
6. Write three numbers which can be divided evenly by 3.

______________ ______________ ______________

7. Write three numbers which can be divided evenly by 9.

______________ ______________ ______________

ACTIVITY 4:

Suggested materials:
workshcet

Subbested strategies:
Level III

These topics were placed at this level because their tests are two fold. i.e. Two criteria for divisibility are needed.
The teacher may need many more examples in these cases before the student is satisfied that the rule works.
Always be sure to present the concept prior to the rule.

A. Divisibility Test for 4.
Criteria 1.--The number must be an even number i. e. 0, 2, 4, 6, 8.
Criteria 2.--The number represented by the last two digits is divisible by 4.

B. Divisibility Test for 6.
Criteria 1.--The number must be an even number.
Criteria 2.--The sum of the digits of the number can be divided by 3.

C. Divisibility for 8.
Criteria 1.--The number is an even number.
Criteria 2.--The number represented by the last three digits is divisible by 8.

Additional material--Divisibility Test for 4, 6, 8.

Level III

1. Can 6 divide 468,765?
2. Can 6 divide 9,726?
3. Can 4 divide 8,500?
4. Can 4 divide 832?
5. Can 8 divide 832?
6. Can 8 divide 5,952?
7. Name a number which can be divided evenly by 8.

   ____

8. Write a number which can be divided evenly by 4.

   ____

9. Write two numbers which can be divided evenly by 6.

   ____________________ ____________________

OBJECTIVES:

1. Given a division problem in which the divisor is a one digit number and which has a remainder, the student can find the quotient.

ACTIVITY 1:

Suggested materials:
worksheet

Suggested strategies:

At this point the student should have a basic understanding of the concept of division.
The teacher should at this time work with the student until he gets in mind that remainders in division problems must be less than the divisor. i.e. If the divisor is 6 possible remainders are 0--problem even or 1, 2, 3, 4, 5.
This helps the student to be watchful of their subtraction.
If in subtracting their result is greater than the divisor, they have not selected the correct multiplication problem. The student then should return to his problem and find a larger product.
The students should be shown that the pattern of checking the problems has now changed, since the product of the quotient and divisor will not be as large as the dividend.
They can then be shown that by adding the remainder to this product they should arrive at the original dividend.

Fractions may be introduced as a method of writing the remainders. i.e. $\frac{\text{remainder}}{\text{divisor}}$

Refer back also to one of the three ways of writing a division problem--that a fraction is also a division problem. Reducing the fraction can be left as an optional activity depending upon the ability of the student.
The worksheets accompanying this activity may be or may not be adequate. Additional worksheets could be given.

ACTIVITY 2:

Suggested materials:
Colored rods as used in other activities.

Directions to the student:
See the worksheet.

Suggested strategies:
This is an activity using manipulative means whereby the student deals with the concrete with the expectation that the student can take this activity and with the help of the teacher, other students or by himself, he can make the transference from the concrete to the intuitive stage of understanding of the concept.
This activity could be used by the teacher on the overhead. The manipulation of these rods shows up very clearly when used in this fashion.

Instructions to the student:
Select a 12 rod.
Select a 10 rod.
Does the ten rod cover the 12 rod? Find a rod you could add to the 10 rod which makes a longer rod covering the 12 rod.
Is it true that a 10 rod plus a 2 rod cover the 12 rod?
You have formed a division problem which looks like this:

$12 \div 10 = 1$ with 2 left over

or

$$\begin{array}{r} 1 \\ 10\overline{)12} \\ \underline{10} \\ 2 \end{array}$$

or $12 = (10 \times 1) + 2$

Try these problems:

1. Select a 12 rod.
   Select two 5 rods.
   Explain what happens when you try to cover a 12 rod with two 5 rods. How much is left over?___________

   Fill in the blanks in this division problem.

$$\begin{array}{r} \square \\ 5\overline{)12} \\ \underline{10} \\ \square \end{array}$$

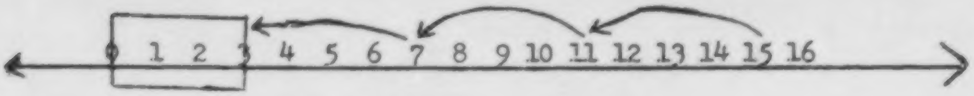
2. Try three other combinations of rods in which some of the original rod is left over. Write your division problems.

ACTIVITY 3:

Suggested materials:
Number lines (1 - 15 or 20)

Suggested strategies:
This activity could be used much like those in Activity 1.

Directions to the student:
On the number line locate 15. Draw a circle around it.
Draw arrows to show how many 4's there are in 15. i.e.



How many 4's are there?____________________________

How many spaces are left over? Draw a box around them.

The number 3 is left over.

Is it true that $(4 \times 3) + 3 = 15$?
A division problem which says the same thing is:

$$\begin{array}{r} 3 \\ 4\overline{)15} \\ \underline{12} \\ 3 \end{array}$$

The three which you found to be "left over" is called the remainder.

Try these problems on a number line and write a division problem.

1. Draw a dot at 20 on the number line.
Draw arrows to show how many 3's there are in 20.

How many 3's are there?
Were there any units "left over?" How many?____________
Write the division problem.

2. Similar problems could be constructed by the teacher for the student.

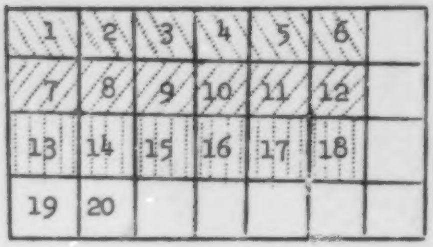
ACTIVITY 4:

Suggested materials:
Grid paper

Directions to the student:

1. Here is a division problem 20 ÷ 6 = . This problem has a remainder.

   On the graph paper, draw a heavy dark line around 20 boxes. Number each box.



1	2	3	4	5	6	
7	8	9	10	11	12	
13	14	15	16	17	18	
19	20					

Take a colored pencil and shade in as many groups of 6 as you can find. How many squares are left over?____ The number of squares left over is the remainder in this problem.

This can also be done by cutting the grid into piences and pasting them into a long line, then fold them every six squares leaving two left (remainder) over.

Activity 5

Worksheet A

Divide and check:

$8\overline{)58}$

$4\overline{)97}$

$7\overline{)786}$

$6\overline{)877}$

$3\overline{)1342}$

$2\overline{)685}$

$9\overline{)31,980}$

$5\overline{)6254}$

$9\overline{)91,577}$

$7\overline{)73,542}$

Activity 5

Worksheet B

Divide and Check:

$5\overline{)48}$

$3\overline{)31}$

$8\overline{)987}$

$6\overline{)465}$

$4\overline{)1727}$

$7\overline{)3580}$

$9\overline{)33,962}$

$2\overline{)46,751}$

$3\overline{)445,986}$

$7\overline{)843,564}$

Activity 5
Bonus Activity

▨	1.	▨	▨	2.	3.	4.	▨	5.	6.	▨	7.
8.		▨	9.				▨	12.		11.	
	▨	12.				▨	13.		▨	14.	
▨	15.				▨	16.		▨	17.		▨
18.				▨	19.			▨	▨	▨	20.
	▨	▨	▨	21.			▨	▨	22.	23.	
▨	24.	25.	26.			▨	27.	28.			
29.					▨	30.				▨	▨

Across

1. 64 ÷ 8
2. 1092 ÷ 3
5. 72 ÷ 2
7. 27 ÷ 9
8. 492 ÷ 6
9. 12,190 ÷ 5
10. 23,248 ÷ 4
12. 29,085 ÷ 7
13. 128 ÷ 2
14. 117 ÷ 3
15. 16,872 ÷ 4
16. 70 ÷ 5
17. 56 ÷ 2
18. 35,441 ÷ 7
19. 5056 ÷ 8
20. 14 ÷ 2
21. 379 ÷ 1
22. 3726 ÷ 6
24. 184,365 ÷ 3
27. 560,169 ÷ 9
29. 12,836 ÷ 1
30. 10,176 ÷ 4

Down

1. 164 ÷ 2
2. 13832 ÷ 4
3. 3175 ÷ 5
4. 288 ÷ 6
5. 1062 ÷ 3
6. 544 ÷ 8
7. 3061 ÷ 9
8. 560 ÷ 7
9. 4226 ÷ 2
11. 690 ÷ 5
12. 1278 ÷ 3
15. 160 ÷ 4
16. 1251 ÷ 9
17. 8 ÷ 4
18. 312 ÷ 6
19. 4725 ÷ 7
20. 1422 ÷ 2
21. 356 ÷ 1
22. 3120 ÷ 5
23. 48 ÷ 2
24. 186 ÷ 3
25. 144 ÷ 8
26. 43 ÷ 1
27. 130 ÷ 2
28. 216 ÷ 9
29. 9 ÷ 9
30. 4 ÷ 2

OBJECTIVES:

1. Given a division problem with a two digit divisor--units digit less than 5, the student can find an answer.

ACTIVITY 1:

Suggested materials:
worksheets

Directions to the student:
The directions are found on the worksheet and in the suggested strategies.

Suggested strategies:
The teacher might choose to go over the worksheet with the students, division problems without remainder, prior to giving the student worksheets A or B. She could use the students' worksheet to show:

1. Division is repeated subtraction--see items 8, 21, 30.
2. The use of a trial divisor--see items 13, 14, 15.
3. Position of numbers in the quotient--see items 4, 5, 17, 18, 26, 27.
4. Check--35, 36.

With repeated usage it could be possible to have students place either no number or an x as in this example in place of the zero's.

```
      192
33 )6336
    33
    303
    297
      66
      66
```

Note: lack of use of zero in this case.

The second worksheet "Division problem with remainder illustrates the same procedure described as in the first one.

1. Division is repeated subtraction--see items 8, 21, 32.
2. The use of a trial divisor--see items 11-16, 24-28.
3. Position of numbers in the quotient--see items 5, 18, 30.
4. Check. 36-39.

These worksheets could also be used as remedial work, or as individually, the teacher could circulate around the room giving needed help. Encourage checking of problems. This seems to be a reinforcement of multiplication.

Activity 6

Worksheet A

Divide and Check:

$10\overline{)40}$

$14\overline{)28}$

$12\overline{)96}$

$63\overline{)189}$

$50\overline{)500}$

$94\overline{)2162}$

$42\overline{)2688}$

$24\overline{)1896}$

$73\overline{)8833}$

$33\overline{)12012}$

Activity 6

Worksheet B

Divide and Check:
Write the remainder as a fraction.

$74\overline{)402}$

$84\overline{)637}$

$32\overline{)156}$

$74\overline{)295}$

$44\overline{)1321}$

$52\overline{)1654}$

$64\overline{)2952}$

$33\overline{)8964}$

$91\overline{)71,432}$

$53\overline{)121,542}$

Activity 6

Worksheet C

Divide and Check:
Write remainders as fractions.

$32\overline{)7648}$

$90\overline{)34,733}$

$63\overline{)189}$

$64\overline{)54,468}$

$81\overline{)96,215}$

$80\overline{)290}$

$64\overline{)45,468}$

$12\overline{)2100}$

$52\overline{)860,236}$

$29\overline{)2117}$

ACTIVITY 2:

$$\begin{array}{rr|c|c}
 & 1 & 9 & 2 \\ \cline{2-4}
33\,\big) & 63 & 3 & 6 \\
 & 33 & 0 & 0 \\ \cline{2-4}
 & 30 & 3 & 6 \\
 & 29 & 7 & 0 \\ \cline{2-4}
 & & 6 & 6 \\
 & & 6 & 6 \\ \cline{3-4}
\end{array}$$

Look at the division problem.
Answer the following questions.

1. Is 6 bigger than 33? ________________
2. Is 63 bigger than 33? ________________
3. How many 33's are in 63? ________________
4. Find 1 in the problem.
5. 1 is worth 100 since 1 is in the hundreds column.
6. 100 x 33 = ________________________
7. Put 3300 under 6636.
8. Subtract.
9. 3036 is the remainder. This is a new division problem.
10. In your new division problem you will find an answer for the tens column.
11. Is 30 bigger than 33? ________________
12. Is 303 bigger than 33? ________________
13. How many 33's are in 303? Ask yourself 30 x 10 = 300, but 10 x 33 = ________________________
14. Is 330 bigger than 303? ________________
15. 10 - 1 = 9   9 x 33 = __________
16. Is 297 bigger than 303? ________________
17. Find 9 in the problem.
18. 9 is worth 90 since it is in the tens column.
19. 90 x 33 = ________________________
20. Put 2970 under 3036.

21. Subtract.

22. 66 is the remainder. This is a new division problem.

23. In this division problem you will find an answer for the one's column.

24. Is 66 bigger than 33? ____________________

25. How many 33's are in 66? ___________ Ask yourself 33 x ____ = 66.

26. Find 2 in the picture.

27. Since 2 is in the ones column.

28. Multiply 2 x 33 = ________________________

29. Find 66 in the problem.

30. Subtract.

31. The remainder is zero. Your problem is finished.

32. 192 is the quotient.

33. 192 x 33 = ______________________________

34. Is your answer in number 33 the same as the dividend? ____________

35. You have just checked your problem.

ACTIVITY 3:

$$\begin{array}{r r|c|c}
 & 1 & 6 & 8 \\ \cline{2-4}
43 \big) & 72 & 4 & 8 \\
 & \underline{43} & \underline{0} & \underline{0} \\
 & 29 & 4 & 8 \\
 & \underline{25} & \underline{8} & \underline{0} \\
 & 3 & 6 & 8 \\
 & \underline{3} & \underline{4} & \underline{4} \\
 & & 2 & 4
\end{array}$$

Look at the division problem.
Answer the following questions.

1. Is 7 bigger than 43? ____________________
2. Is 72 bigger than 43? ____________________
3. How many 43's are in 72? ____________________
4. Find 1 in the problem.
5. 1 is worth 100 since 1 is in the hundreds column.
6. 100 x 43 = ______________________________
7. Put 4300 under 7248.
8. Subtract.
9. 2948 is the remainder. This is a new division problem.
10. In your new division problem you will find an answer for the tens column.
11. Is 29 bigger than 43? ____________________
12. Is 294 bigger than 43? ____________________
13. How many 43's are in 294? Ask yourself: 40 x 7 = ______________
    43 x 7 = ______________
14. Is 301 bigger than 294? ____________________
15. What is 43 x 6? ________________________
16. Is 258 bigger than 294? ____________________
17. Find 6 in the problem.
18. 6 is worth 60 since it is in the 10's column.
19. 60 x 43 = ______________________________

20. Put 2580 under 3036.

21. Subtract.

22. 368 is the remainder. This is a new division problem.

23. In this division problem you will find an answer for the one's column.

24. Is 36 bigger than 43? ____________________

25. Is 368 bigger than 43? ____________________

26. How many 43's are in 368? Ask yourself: 40 x 9 = ______________
43 x 9 = ______________

27. Is 387 bigger than 368? ____________________

28. What is 43 x 8? __________________________

29. Is 344 bigger than 368? ____________________

30. Find 8 in the problem.

31. 8 is worth 8 since it is in the one's column.

32. Put 344 under 368.

33. Subtract.

34. 24 is the remainder.

35. Is 24 bigger than 44? ____________________

36. You problem is finished, the quotient is 168 24/43.

37. 168 x 43 = ______________________________

38. 7224 + 24 = _____________________________

39. Is the answer in number 38 the same as the dividend? ______________

40. You have just checked your problem.

OBJECTIVES:

Given a division problem with a two digit divisor--units digit equals 5, the student will find an answer.

ACTIVITY 1:

Suggested matherials:
worksheet

Suggested strategies:
Follow the procedure of Activity 6 if students are still having trouble.

On the accompanying worksheet the odd numbered problems are even. The even problems have remainders.

This sheet could be used as a timed drill or assigned as group activity. One effective device to get good participation in the group is for the teacher to announce that everyone in the group will receive the same grade. Establish some ground rules. All members of the group must work the problems and check. They then check their work by comparing answers. They turn in one sheet of answers with the names of the members of the group on them. This helps in recording grades. This type of activity creates an atmosphere of competition in the classroom. The students have a built in check system. It also seems to help their self confidence.

Activity 7

Worksheet A

Divide and check:

Write each remainder as a fraction:

1. $25\overline{)125}$

2. $15\overline{)226}$

3. $45\overline{)1775}$

4. $85\overline{)19{,}654}$

5. $65\overline{)1560}$

6. $95\overline{)101{,}764}$

7. $55\overline{)13{,}464}$

8. $75\overline{)17{,}655}$

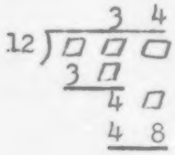
Activity 7

Worksheet B

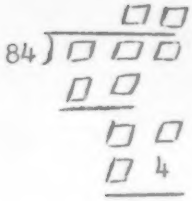
Bonus:

Put a number in the box.

1.



2.



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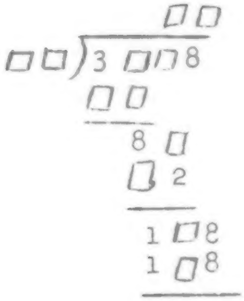


OBJECTIVE:

Given a division problem with a two digit divisor--units digit greater than 5, the student will find an answer.

ACTIVITY 1:

Suggested materials:
worksheet

Suggested strategies:

The strategy for this activity follows from Activities 6 and 7.

Problems in worksheet A have no remainders.
Problems in worksheet B have remainders.
Problems in worksheet C contain both problems with remainders and those without.

It would be possible to continue this unit into division by a three digit number if desired by the teacher.

Activity 8

Worksheet A

Divide and check:

$19\overline{)76}$ $48\overline{)336}$

$96\overline{)864}$ $59\overline{)354}$

$18\overline{)612}$ $97\overline{)4462}$

$76\overline{)1444}$ $79\overline{)1975}$

$48\overline{)10{,}848}$ $88\overline{)11{,}792}$

Activity 8

Worksheet B

Divide and Check:
Write remainder as a fraction.

$56\overline{)115}$ $68\overline{)315}$

$38\overline{)1675}$ $17\overline{)4652}$

$76\overline{)2572}$ $28\overline{)5689}$

$88\overline{)16,555}$ $49\overline{)97,642}$

$96\overline{)112,352}$ $121\overline{)79,523}$

Activity 8

Worksheet C

Divide and Check:
Write the remainder as a fraction.

$57\overline{)268}$ $36\overline{)288}$

$69\overline{)897}$ $66\overline{)792}$

$19\overline{)1235}$ $79\overline{)3469}$

$87\overline{)63{,}962}$ $46\overline{)15{,}732}$

$27\overline{)66{,}555}$ $98\overline{)121{,}475}$

Bonus: Numbers are missing in the following problems. Find hem.

```
24  3695             56  79,562           25  60295
    24                   56                   50
    129                  235                  102
    120                  224                  100
      95                  118                   29
      72                  112                   25
      23                    62                   45
                            56                   25
                             6                   20
```

COMPETENCY: Solve additions and subtraction problems having denominators less than 20.

OBJECTIVE: Given a fractional addition or subtraction problem having denominators less than 20, the student will be able to solve it.

ACTIVITY 1: Fraction Pies

Suggested Material: aluminum pie plates, colored construction paper or or oaktag
On bottom of each plate, mark a fractional equivalent of 1 (1/1, 2/2, 3/3, etc.) Cut out circles of colored paper, (at least two of each color) each large enough to cover the bottom of a plate. Leave the blue circles alone. Cut the yellow circles into halves, and mark 1/2 on each piece. Cut the green circles into thirds, and mark 1/3 on each piece. Continue in the same way for 4ths, 6ths, 8ths, 9ths, 12ths, 18ths, or until you run out of colors.

Directions to Students: (See worksheets labeled Activity 1-A, 1-B, 1-C.)

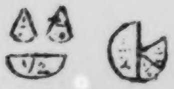
Suggested Strategies:

1. Make sure you have enough kinds of fractions to answer the problems (i.e., say 4 halves, 8 fourths, etc., if any answer comes out in 12ths, there should be a pie cut up into twelfths).
2. This can be worked with the given worksheets, or with problems from the students' books.
3. Although this is designed for individuals or very small groups, it may be best to do the 1st few examples with the class as a whole.

*4. This is intended to be supplemental material only, and was designed primarily to give students the "feel" of fractions, so that they would not make the error of 2/3 + 1/4 = 3/7.

5. Do not give all three worksheets at once, so as to keep student interest up.
Activity 1-A is adding fractions whose sum is less than one.
Activity 1-B is adding fractions whose sum is more than one.
Activity 1-C is subtracting fractions.

ACTIVITY 1-A:



Directions to Student:
These fraction pies are to help you add fractions.
To add 2/6 + 1/2, take two pieces with 1/6 on them (for 2/6),
and one piece with 1/2 on it. (for 1/2)
Put all three pieces together.
Find the smallest number of pieces of the same color that will exactly cover the pieces you have.

Example: to cover 2/6 + 1/2, you need five 1/6 pieces,
so your answer is 2/6 + 1/2 = 5/6.

Try these problems:

(a) 1/3 + 2/4 =

(b) 2/9 + 7/9 =

(c) 1/6 + 2/6 =

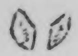
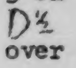
(d) 1/2 + 1/6 =

(e) 3/4 + 1/6 =

(f) 1/3 + 1/4 =

ACTIVITY 1-B:

Directions to Student:

These fraction pies are to help you add fractions. Sometimes when you add fractions, your answer is more than one whole.
To add 2/3 + 1/2, take two pieces with 1/3 on them (for 2/3) 
and one piece with 1/2 on it (for 1/2) 
Put all three pieces together (they will over lap)

To find the answer, think:
How many whole pies could I make out of the pieces? (One)
How much pie do I have left over? (Find the smallest number of pieces of the same color that will exactly cover the part that overlaps.)
Example: To cover 2/3 + 1/2, I need one whole pie plus one 1/6 piece, so my answer is 2/3 + 1/2 = 1 + 1/6 = 1 1/6

Try these problems:

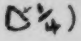
(a) 1/2 + 3/4 = (b) 3/8 + 7/8 = (c) 5/6 + 1/4 =

(d) 1/3 + 2/3 = (e) 5/8 + 1/2 = (f) 7/9 + 2/3 =

ACTIVITY 1-C:

Directions to Student:

These fraction pies are to help you subtract fractions.
To subtract: 3/8 - 1/4
Take as many pieces of the same color as you need to make the first number, and put them together (three 1/8 pieces)

Take as many pieces as you need to make the second number (one 1/4 piece ) and put it on top of the first number.

Your answer is the smallest number of pieces of the same color that you need to finish covering the first number.

Example:
to finish covering 3/8 - 1/4, you need one 1/8 piece.
Your answer is 3/8 - 1/4 = 1/8

Try these problems:

(a) 1/2 - 1/4 = (b) 5/6 -1/3 = (c) 2/3 - 2/9 =

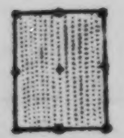
(b) 2/3 - 1/9 = ([illegible]) 1/2 - 1/3 = (f) 3/4 - 2/3 =

ACTIVITY 2: Addition and Subtraction of Fractions

Suggested Materials: worksheet

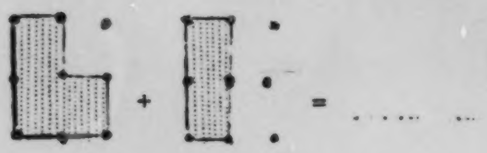
Directions to Student:

Here is one unit of area



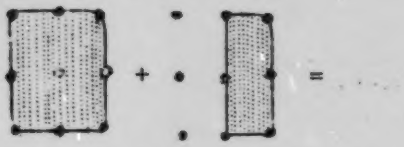
Fill in the fraction of the part shaded below each figure.
Write the sum of the two fractions after the = sign.
The first two have been done for you.

(1)



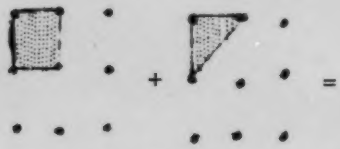
3/4 + 2/4 = 1 1/4

(2)



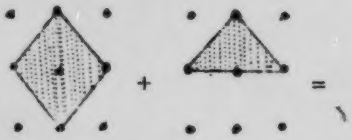
______ + ______ = ______

(3)



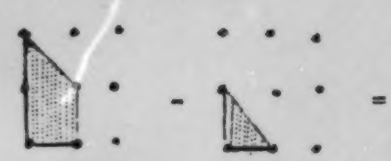
______ + ______ = ______

(4)



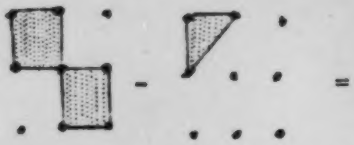
______ + ______ = ______

(5)



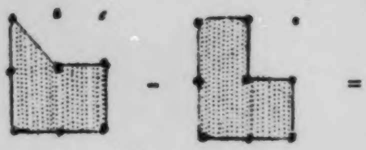
3/8 - 1/8 = 2/8

(6)



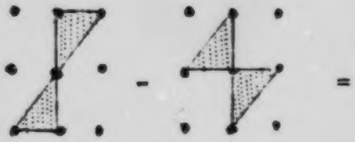
______ - ______ = ______

(7)



______ - ______ = ______

(8)



______ - ______ = ______

Suggested Strategies:

This has been drawn on paper, but the idea could be used with geo-boards or in group discussion with overhead transparencies.

ACTIVITY 3: Bingo

Suggested Materials: A set of 20 cards marked with the following problems, one problem to a card. The answers (in parentheses) may be marked on the back of the cards, if desired.

1/3 + 1/3	(2/3)	2/5 + 1/4	(13/20)
3/8 + 4/8	(7/8)	1/6 + 7/12	(3/4)
3/4 - 1/4	(1/2)	3/5 + 4/15	(13/15)
5/6 - 4/6	(1/6)	3/4 - 2/3	(1/12)
1/6 + 3/4	(11/12)	4/9 - 1/3	(1/9)
3/5 + 1/4	(17/20)	9/10 - 1/2	(2/5)
1/3 + 2/5	(11/15)	1/2 - 1/12	(7/16)
1/4 + 3/10	(11/20)	5/6 - 4/9	(7/18)
1/2 + 4/9	(17/18)	1/6 + 2/3	(5/6)
3/8 + 5/16	(1/16)	5/6 - 1/2	(1/3)

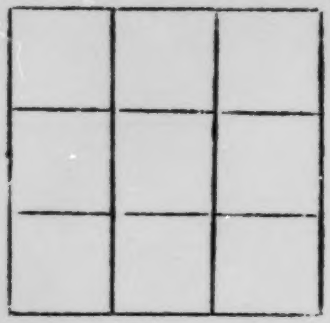
One card marked with all the answers,

1/2, 1/3, 2/3, 3/4, 2/5, 1/6, 5/6, 7/8, 1/9, 1/12, 11/12, 11/15, 13/15, 7/16, 11/16, 17/18, 7/18, 11/20, 13/20, 17/20.

Label it: ANSWER CARD

Directions to Student:

1. Draw a grid of 9 squares on your paper.



2. Take the answer card, and pick any nine fractions from it. Write them in any order on your grid. (Just be sure to put only one fraction in each box.)
3. Shuffle the problem cards, and put them face up. When everyone is ready, cut the deck.

4. Work the addition or subtraction problem and check your grid for the answer. If it appears, cross it out.
5. Put the top card aside (without turning it over) and work the next problem, looking for the answer.
6. The winner is the person who gets three crosses in a row (Down, across, or diagonally). Check his answers by looking at the cards you put aside (the answers are on the back). Make new grids and start again.

Suggested Strategies:

This activity can be varied in innumerable ways. The class may do it as a whole, with the teacher turning over the cards. The grid could be changed, the problems and answers could be changed, the grids could be dittoed off, there could be a time limit, etc..

We suggest that the teacher go through it with the class, and show the students how to fill in the grid. Also, if you do it with a group, it would be helpful to have the "answer card" on an acetate for the overhead.

ACTIVITY 4:

Suggested Materials: worksheet

Directions to Student:

Place either + or - in each empty space to make the equations true. You can read up or down or across.

3		2	=	5
	▨		▨	=
8		1		9
=	▨	=	▨	
11		3	=	14

5/6		$\frac{1}{4}$	=	7/12
	▨		▨	=
1/3		5/12		3/4
=	▨	=	▨	
$\frac{1}{2}$	=	2/3		1/6

ACTIVITY 5:

Suggested Materials: egg carton, marbles, cubes or other small objects

Directions to Student:

You will be using an egg carton and some marbles to add and subtract fractions. The rule is that only one marble can go in one space.

(+) To add: Fill one half of the spaces with marbles, and then fill 1/4 of all the spaces.

What fraction of all the spaces are filled? __________

So 1/2 + 1/4 = ____________

Using the egg carton, try these problems.

(a) 1/4 + 1/3 = ______ (c) 2/3 + 1/4 = ______

(b) 1/2 + 1/3 = ______ (d) 1/12 + 3/4 = ______

If all your answer denominators are 12, can you find simpler fractions for some of them?

(-) To subtract: Fill one half of the spaces with marbles, and then take out marbles from 1/4 of all 12 spaces. (First you filled 6 spaces, and then emptied 3 spaces) What fraction of all the spaces are still filled? ________
So 1/2 - 1/4 = ________.

Using the egg carton, try these problems.

(a) 1/3 - 1/4 = ______ (c) 2/3 - 1/4 = ______

(b) 1/2 - 1/3 = ______ (d) 3/4 - 1/12 = ______

Now try these problems. Look at the signs to see if you add or subtract.

(a) 1/2 + 1/12 = _____ (e) 1/2 - 1/12 = ______

(b) 1/4 + 5/12 = ______ (f) 5/12 - 1/4 = ______

(c) 7/12 - 1/12 = ______ (g) 7/12 + 1/12 = ______

(d) 2/3 - 1/12 = ______ (h) 2/3 + 1/12 = ______

Suggested Strategies:

If student's reading level is low, read instructions aloud. This activity may be varied to include just addition, or just subtraction, or even for learning the size of fractions ([illegible] (is 1/3 > 1/6, etc.)

COMPETENCY: The subtraction of common fractions

OBJECTIVES: Given any combination of fractions, mixed numbers, and whole numbers to perform the required subtraction

ACTIVITY 1: To subtract fractions with common denominators

Suggested materials:

Strips of graph paper (4 squared per inch), 5 sq/inch, 6 sq/inch, 8 sq/inch, and 10 sq/inch; rulers; yardsticks; meter sticks.

Directions to student:

On the graph paper with the largest squares, measure one inch. How many squares in one inch? in 3/4 inch? in 1/4 inch? in 1/2 inch? How many squares are left when we subtract 3/4 - 1/4? What fraction is this?

Now take the graph paper with 6 sq/inch and mark off rows of two squares, of three squares, of four squares, and of five squares. What fractions of an inch are represented by the rows of squares?

(yes/no) 1/6"________; 1/3"________; 2/6"________;

5/6"________; 2/3"________; 1/2"________;

2/5"________; 4/6"________.

What fraction is the answer to the following:

2/3 - 1/6 = ________

5/6 - 3/6 = ________

4/6 - 2/6 = ________

Suggested strategies:

Continue this with the other strips of graph paper.

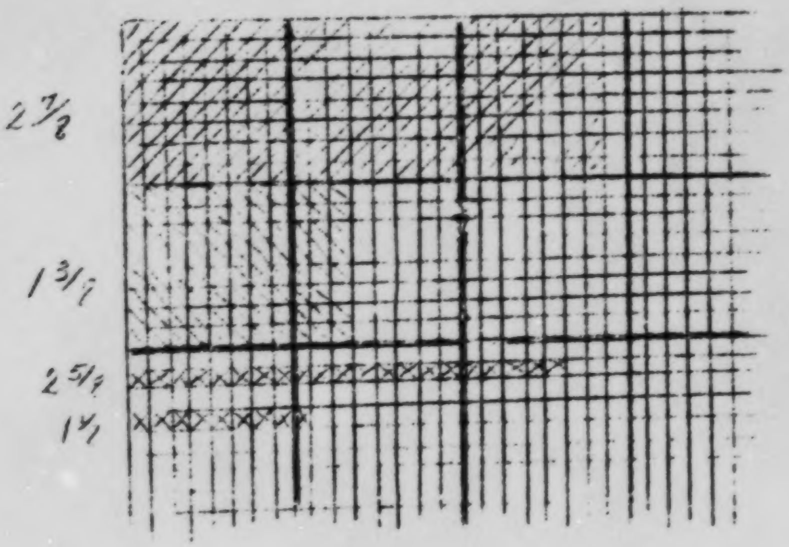
ACTIVITY 2: To subtract mixed numbers with common denominators

Suggested materials:

Same as Activity 1, plus colored pencils

Directions to student:

Use the 8 square graph paper. Measure a row of squares 2 ½ inches in length. How many squares are there?
How many squares are in one inch?
Can you draw a one-inch square?
Now using one-inch squares show 2 7/8 inches.
Do the same thing below this to show 1 3/8.
What is the answer to 2 7/8 - 1 3/8?
Use a row of squares to represent 2 5/8 and use another row of squares to represent 1 1/8.
What is the answer to 2 5/8 - 1 1/8?



Suggested strategies:

Use strips of the other graph paper to demonstrate the subtraction of other fractions.

ACTIVITY 3: To subtract a fraction from a whole number

Suggested materials:

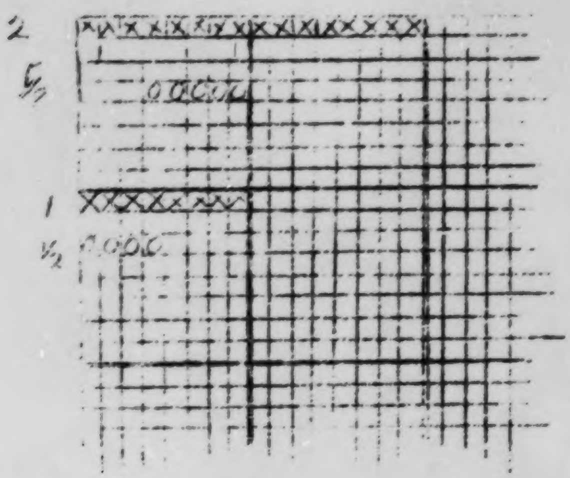
Same as for Activity 1 and 2.

Directions to student:

Start with the 8 square graph paper.

Mark off a row of two inches. Below this mark off a row of 5/8 inches.
What is the result of 2 - 5/8?

Now mark off a row of one inch. Below this mark off a row ½ inch.
What is the answer to 1 - ½?



Suggested strategies:

Work other combinations using some of the other strips of graph paper.

COMPETENCY: Subtraction of decimals

OBJECTIVES: Given any combination of decimals and whole numbers to perform the required subtractions.

ACTIVITY 1: Using graph paper to show the subtraction of decimals such as .8 - .5, .35 - .21, and .248 - .132.

Suggested materials:

Meter stick, ruler in tenths, strips of graph paper in millimeters, 10 sq/inch, and 20 sq/inch.

Directions to student:

Using the meter stick have the student read the number of centimeters on the stick and the number of millimeters in a centimeter. Have them point out the position of such points as 8 cm, 12.5 cm, 6.9 cm, 21.2 cm, .5 cm.

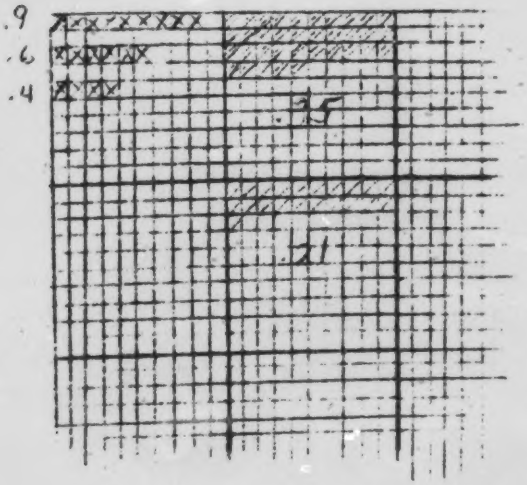
Now use a strip of the 10 square graph paper. Have the student mark off rows of squares .4", .6", and .9" in length. Ask for the answers for these subtractions:

.6 - .4 = ________ .9 - .6 = ________ .9 - .4 = ________

Next have the student mark off 35 squares in one of the one-inch squares and 21 squares in an ajoining square.

Ask: Does 35 squares equal .35?
Does 21 squares equal .21?
What is the answer to .35 - .21?

Now give the problem. $\begin{array}{r} .548 \\ -\ \underline{.132} \end{array}$ What is the answer?



Suggested strategies:

Try some subtractions without graph paper such as:

.8	.4	.78	.56	.481	.643
-.5	-.1	-.52	-.39	-.123	-.291

If there is any trouble with these problems, more work with the graphs should help.

COMPETENCY: Subtraction of Decimals

OBJECTIVES: To subtract decimals from whole numbers such as 2 - .6, 1 - .42, 3 - .645

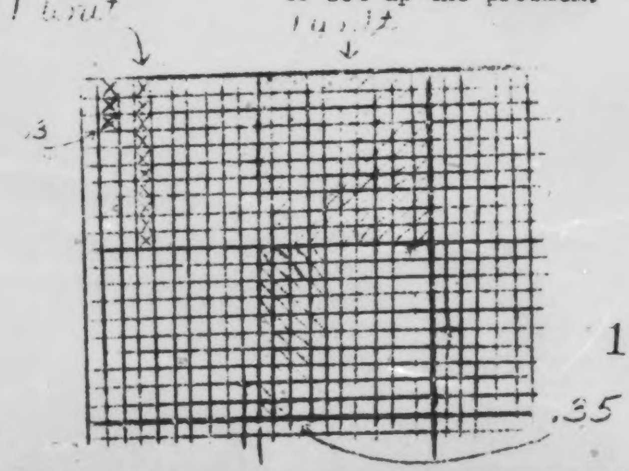
ACTIVITY 2: Using graph paper or other devices to subtract decimals from whole numbers

Suggested materials:

meter stick, ruler in tenths, strips of graph paper in millimeters, 10 sq/inch, and 20 sq/inch.

Directions to student:

10 cm will be used to represent 1 unit.
Use the meter stick to locate 10 cm on the stick as 0 - 10 or 20 - 30, etc. Can you give the answer to 1 - .3? yes/no
If no, have the student mark off a row of 10 squares and below it a row of 3 squares. Then repeat the question 1 - .3 = ________ If yes, use the meter stick, point out the number of cm.
Ask: What is 1 - .35?
What is 1 - .72?
If these are not answered correctly go to the graph paper to set up the problem.



the use of 10 sq. graph paper

Subtracting a 3 place decimal from a whole number should follow without further work with graphs. Use of the 20 sq. graph paper will give more variety.

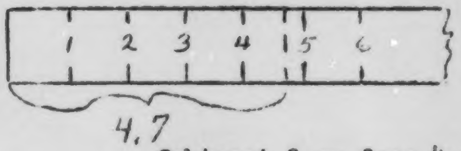
COMPETENCY: Subtraction of decimals

OBJECTIVES: To subtract decimals such as 4.8 - 2.3, 2.53 - 1.25, and 3.785 - 1.362.

ACTIVITY 2: The use of meter sticks and graph paper to subtract decimals.

Suggested materials:
Meter sticks, graph paper in millimeters, 10 sq/inch, and 20 sq/inch.

Directions to student:
Use the meter stick to show 4.7 - 2.4
First locate 4.7 cm on the meter stick.



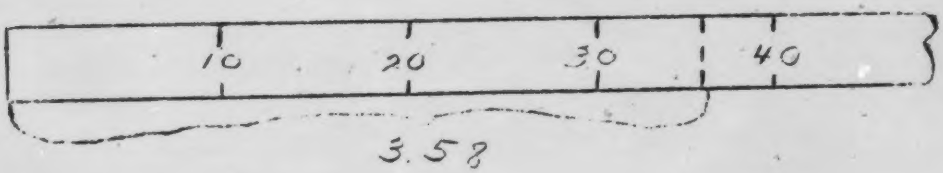
4.7
- 2.4

Subtract 2 cm from 4 cm, leaving ____________________.

Then subtract .4 cm from .7 cm, leaving _____________.

The answer to the problem 4.7 - 2.4 is _____________.

Use the meter stick to show 3.58 - 1.25



Subtract 1 from 3 leaving __________________________.

Subtract .25 from .58 leaving ______________________.

The answer to the problem 3.58 - 1.25 is __________.

Other problems can be used with the graph paper.

COMPETENCY: Recognition of mathematical phrases. Construction of verbal phrases from symbols, and phrases involving symbols from verbal ones. This competency is necessary for work with mathematical sentences.

OBJECTIVES; The student will recognize mathematical phrases and be able to translate verbal phrases into those involving symbols and vice versa. Student will give examples of mathematical phrases orally and in writing. Student will produce phrases using symbols, having been given word phrases and vice versa.

ACTIVITY 1: Words from symbols

Suggested Material:

Transparency on which are listed phrases using symbols. Example:

1. $2 + 3$
2. 7×4
3. $2(8 + 5)$
4. $\frac{2 + 4}{3}$
5. $8 - 5$
6. $7 - (2 + 3)$
7. $7 + \{2(6 + 1)\}$
8. $9 + \frac{6}{2}$

Directions to Student:

Read the phrase, using a word or words for the symbols you see.

Suggested Strategies:

Symbols of inclusion may need to be reviewed if students hesitate in reading quantities in parentheses, brackets or in numerator of fraction.

ACTIVITY 2: Symbols from words

Suggested Material:

Prepare a transparency giving verbal phrases in a column. Students will insert phrase in symbols opposite the phrase on the transparency. All other phrases on the transparency will be kept covered except the one being worked on so that attention of all students will be on the phrase being considered.

Sample Transparency

Verbal phrase	Phrase using symbols
Two plus four	____________
Seven minus one	____________
Two less than five	____________
Eight times two	____________
Six less four	____________
Twice the sum of two and three	____________
One-half of the sum of three and five	____________
The sum of three and five less one	____________
Divide the sum of three and five by three	____________

Directions to Student:

Insert a phrase on the transparency opposite the given verbal one, using symbols for each verbal phrase.

Suggested Strategies:

Instead of using a transparency, the phrases may be given orally by the teacher, or by a student. Students designated would then come to the board to write the phrase in symbols.

ACTIVITY 3. Symbols-from-Words Baseball

Suggested Materials:

Each student prepares five verbal phrases, writing them on a card or sheet of paper. The class is divided into two teams, both of which select a pitcher. A coin is flipped to determine the team at bat. Pitchers collect papers containing verbal phrases from team members. Places in the classroom are designated 1st base, 2nd base, 3rd base, and home plate. A member of the team at bat receives orally from the opposing pitcher a verbal phrase. He must write it on the board in symbols. If it is correct, he advances to 1st base; if not, that is one out. Teacher may act as umpire and scorekeeper. Play continues until team has three outs. Then opposing team comes to bat and process is repeated. Number of innings to be played can be determined according to time available.

Directions to Student:

Prepare a list of five mathematical phrases using words. Example: 'two times the sum of three and eighteen." Give your list to your pitcher. Pitcher will use the phrases given him by his team to give to the opposing batter. The batter will go to the board and write the phrase using symbols. Example "2(3 + 18)". If correct, he goes to 1st base. If not, that is an out. The next batter comes up. The pitcher gives him a phrase. He writes it on the board. If correct, he takes 1st base; the man on 1st goes to 2nd base. Play proceeds as in ordinary baseball.

Suggested Strategies:

One way of dividing group into two teams is to have the boys on one team and the girls on the other. It is better not to allow students to choose teams as the poorer students will not be chosen until the very end.

COMPETENCY: Construction of mathematical sentences from phrases.

OBJECTIVE: The student will construct mathematical sentences using phrases combined with symbols for equality and inequality.

ACTIVITY 1: True Sentences from Phrases

Suggested Materials:
Cards on which have been written five mathematical phrases. Each student receives a card giving him five phrases. He must put each phrase into two true sentences, using symbols of equality or inequality.

SAMPLE CARD

Phrases	Sentences
$2 + 3$	Student might fill in: $2 + 3 = 5$ $2 + 3 \neq 7$
$3(6 + 4)$	$3(6 + 4) = 30$ $3(6 + 4) > 10$
$\frac{4 + 2}{3}$	$\frac{4 + 2}{3} = 2$ $\frac{4 + 2}{3} < 6$
$8 - (4 + 2)$	$8 - (4 + 2) < 8$ $8 - (4 + 2) \neq 8$
$\frac{1}{2}(4 + 2)$	$\frac{1}{2}(4 + 2) = 3$ $\frac{1}{2}(4 + 2) \neq 4$

Directions to Student:
Make two mathematical sentences from each of the phrases on this card. Your sentences must be true. Use symbols such as $=$, $\neq$, $<$, $>$, $\leq$, $\geq$ in your sentences. Do not write on this card.

Suggested Strategies:
Activity may be used as a game, with the student finishing first receiving a point if all his sentences (read orally) are correct. Students then exchange cards and begin again. Also, students might work in teams of two or three to complete the card.

ACTIVITY 2: False sentences from phrases

Suggested Materials:
Cards containing five mathematical phrases. Each student receives a card giving him five phrases. He must put each phrase into two false sentences, using symbols of equality or inequality.

SAMPLE CARD

Phrases	False Sentences
$2 + 3$	Student might fill in: $2 + 3 < 5$ $2 + 3 \neq 5$

(Teacher will supply four additional phrases.)

Directions to Student:

Make two mathematical sentences from each of the phrases on this card. Your sentences must be false. Use symbols =, ≠, <, >, ≤, ≥ . Do not write on this card.

Suggested Strategies:

This activity may be used as a game, with the student finishing first receiving a point if all his sentences (read orally) are correct, i.e. false. Students then exchange cards and begin again. Also, students might work in teams of two or three to complete the card.

ACTIVITY 3: "Heads Up" Game. (See Marie's Educational Materials.)

Suggested Materials:

To make your own game, prepare a card for each student, or have the students put this outline on a sheet of paper.

☐ + ☐ = ☐
☐ - ☐ = ☐
☐ X ☐ = ☐ ± ☐
☐ - ☐ = ☐ ± ☐

Write the integers from 0 to 20 on the faces (one on each face) of 14 cubes. Repetition of integers will occur. Place the 14 cubes in a box, shake and throw them out on the desk. Read and list, on the board or overhead, the 14 numbers appearing on the faces turned up. If any number is repeated, list it as many times as it is turned up. Students must insert these integers into the blanks to construct true sentences. At the end of a specified time, depending upon the ability of the students and their familiarity with the game, students total the numbers they have used to construct true sentences and subtract from the total any numbers not used. Result is their score for that game. If cubes are not available, 14 numbers could be drawn from a box containing 3 or 4 sets of integers from 0 to 20 written on small cards. Also, fractions or decimals could be used with the integers. Another variation might be to change the = to ≠, <, >, ≤ , ≥ .

Directions to Student:

Fill in the blank squares with the numbers turned up on the cubes, as listed on the board, to make each of the four open sentences on your card true. At the end of three minutes we will stop and you will total the numbers you have used to make true sentences. From that total you must subtract any numbers you did not use. The result will be your score.

Suggested Strategies:

Depending upon the size of the class or group using this game, a method of checking to be sure sentences constructed are correct and score is correctly computed will be necessary. A student may be appointed to check the sentences of a small group, or sentences might be read aloud, put on the board and checked by the entire group.

COMPETENCY: Construct mathematical sentences from given verbal problems; translate mathematical sentences into verbal problems.

OBJECTIVE: The student will be able to write open sentences given certain conditions to be met, i.e. a verbal sentence or problem.

Given an open sentence, the student will be able to make up a verbal problem to fit it.

ACTIVITY 1. Matching Sentences

Suggested Materials:

Cards on which verbal sentences are written in column form with corresponding open number sentences opposite, but not in the same order. Students must find the number sentence which corresponds to each verbal sentence.

SAMPLE CARD

Verbal Sentences	Number Sentences
1. Twelve times a number is less than seven.	1. $x - 43 = 20$
2. Four times some number is eight.	2. $\frac{1}{2}(x + 7) = 6$
3. Two less than a number is nine.	3. $12x < 7$
4. One-half the sum of a number and seven is six.	4. $2x < x + 9$
5. A number divided by eight is less than six.	5. $4x = 8$
6. A number less four equals three.	6. $4x - 5 = 7$
7. A number minus forty-three equals twenty.	7. $x - 2 = 9$
8. Five less than four times a number is seven.	8. $\frac{x}{8} < 10$
9. The product of a number and two is less than the sum of that number and nine.	9. $6x > 50$
10. Six times a number is greater than fifty.	10. $x - 4 = 3$

Directions to Student:

Match each verbal sentence with a number sentence as shown in the completed sample.

ACTIVITY 2. Make up a verbal problem to fit the sentence.

Suggested Materials:

Cards on which one or more open sentences are listed. Student must compose a verbal problem which could be translated into the given open sentence. For example, if one of the sentences on his card were: $4x = 8$, student might respond: The perimeter of a square is eight inches. How long is each side?

Directions to Student:

Given an open sentence, you must compose a verbal problem to fit it.

Suggested Strategies:

This activity can be done orally, with teacher giving each student, in turn, an open sentence.

Activity can be used as a game, with the first student correctly fitting a verbal problem to the open sentence receiving a point.

Students might write an open sentence and challenge the group to find a verbal problem to fit it. Student who does scores a point.

COMPETENCY: Solve simple linear equations.

OBJECTIVE: The student will transform open sentences into true sentences without using the axioms. (Open sentences will be simple enough so that students will be able to find solution by inspection.)

ACTIVITY 1. Secret Number Sentence

Suggested Materials:

A transparency on which are listed ten or more open sentences. In this game the secret number is represented by a square or triangle. The secret number is that which will make the open sentence true.

SAMPLE TRANSPARENCY

1. $\square + 3 = 7$
2. $\triangle + 3 - 1 = 6$
3. $\triangle - 4 = 4$
4. $4\square = 4$
5. $\triangle \div 4 = 5$
6. $\frac{\triangle}{5} = 4$
7. $(5 + 3) = 16$
8. $\triangle = \triangle$
9. $5(\square - 2) = 5$
10. $3\triangle = 15$

Directions to Student:

Give the number which will make the open sentence true. If more than one number will make it true, indicate the numbers which may be inserted to make a true sentence.

Suggested Strategies:

Note that Exercise No. 8 above is an identity. In Exercises Nos. 4 and 10, the indicated multiplication may need to be pointed out to students.

An "x" may be inserted in the sentences instead of the square or triangle.

Instead of using a transparency, sentences could be listed on cards. Students could then copy the sentences, inserting the number which makes it true. First student to finish his card correctly receives a point. Students then exchange cards and begin again.

ACTIVITY 2. Secret-number-sentence Baseball

Suggested Materials:

Cards on which have been written five open sentences. Class or group is divided into two teams. Each participant is given a card and directed to find the number or numbers which will make his open sentences true. The first student to finish his sentences correctly goes to 1st base. (Bases and home plate will be located at specified places in the classroom.) His team is then automatically the team at bat. Students exchange cards. If the first student to finish his card correctly this time is on the team at bat, he goes to first base. Student on 1st goes to 2nd. However, if first one finished is on the opposing team, that counts as one out for the team at bat. Play continues until agreed-upon number of innings have been completed.

Directions to Student:

On a separate sheet of paper copy the . sentences on your card. Insert, in the square or triangle, the number which will make the sentence true. Raise your hand when you finish.

Suggested Strategies:

All students must stop working when someone raises hand. Teacher may say "Stop !". Checking may be done orally.

This activity may be varied by designating certain cards which list more difficult sentences as "double" or "triple" or, if sentences are of great difficulty, "homerun."

ACTIVITY 3. Secret-number-sentence Baseball (using students as pitchers to give open sentences)

Suggested Materials:

Cards on which are written one or more open sentences. Class or group is divided into two teams, each of which selects a pitcher. Open-sentence cards are divided between the two pitchers. First student at bat receives an open sentence from the pitcher, i.e. "$2x = 10$." Student must give value for x which makes the sentence true. If he does so correctly, he goes to 1st base. If his reply is incorrect, a student from the opposing team must give the correct answer. If he does, student at bat is out. If the answer given by the opposing team member is incorrect, student at bat goes to 1st base on an error. Play continues until team at bat has three outs. Then opposing team comes to bat and play continues as indicated above. Teacher, or appointed student, keeps score and acts as umpire. Game ends after agreed-upon number of innings.

Directions to Students:

Class will be divided into two teams: girls on one team; boys on the other. Each team will choose a pitcher whose task will be to give open sentences to the opposing team for solution. Students will come up to bat in the order in which seated. If student gives correct solution, he will go to 1st base. If his answer is incorrect, someone from the other team must give the correct solution. Pitcher will call on someone from his team to do this. If the answer from the opposing team member is correct, that will count as an out for the team at bat; if it is incorrect, the student at bat will go to 1st base on an error. All correct answers are singles and advance student on base to the next base.

Suggested Strategies:

Teacher may ask students to compose the open sentences to be used. They would then give their sentences to their pitcher to use against the opposing team.

More difficult sentences may be marked as a double, triple or homerun.

For use with a unit on verbal problems, cards might give such problems which would need to be translated into open sentences by the students and then solved.

Cards might list some open sentences and some verbal problems, with the verbal problems designated "triple" or "homerun."

COMPETENCY: Solve linear equations through use of the axioms.

OBJECTIVE: The student will find an operation which undoes a given one.

ACTIVITY 1. Doing and Undoing

Suggested Materials:
Prepare a list of operations for which an UNDOING OPERATION can be found. Such a list might be put on a transparency, on the board, or on individual cards which could be given students.

SAMPLE LIST

OPERATION	UNDOING OPERATION
1. Turning on the TV set	
2. Putting on my shoes	
3. Walking home from school	
4. Earning $5.00	
5. Wrapping a package	
6. Losing 5 yards in a football game	
7. Adding 10 to a number	
8. Walking up the stairs	
9. A rise of 10° in temperature	
10. Subtracting 5 from a number	

Directions to Student:
Give the operation which undoes the listed one.

Suggested Strategies:
This can be done orally, with operations given by teacher and responses by students. It can be a written activity, with students writing the undoing operation on a sheet of paper.

ACTIVITY 2. Undo this.

Suggested Materials:
Each student receives a card which lists one or more operations, mathematical or otherwise. Activity 1 above lists some operations which could be used. Each student takes a turn to act out the operation on his card. For example, if his operation is: Adding 5 to a number, a student might collect five sticks of chalk and place them, one at a time on the tray with the rest of the chalk. The first student to raise his hand and give the undoing operation correctly earns a point.

Directions to Student:

Each of you will take a turn coming to the front of the room and acting out one of the operations on your card. The other students will try to think of an operation which undoes the one you are illustrating. Raise your hand when you think you know what is being illustrated and can give the undoing operation. You will receive a point for every correct answer.

Suggested Strategies:

In listing operations on cards, be certain they are of the type that can be acted out easily with articles available in the classroom. Include some involving addition and subtraction since the purpose of the activity is to reinforce the fact that addition undoes subtraction and vice versa.

If you wish to avoid the subtraction axiom and use only the addition axiom, substitute ADD THE OPPOSITE for SUBTRACT.

COMPETENCY: Solve linear equations through use of the axioms.

OBJECTIVE: Students will be able to use the multiplication axiom in solving equations.

ACTIVITY 1. Suggest a Solution.

Suggested Materials:

Prepare a list of simple verbal problems which will require use of the multiplication axiom in their solution. Students will suggest a method of solution.

SAMPLE SENTENCE	ACCEPTABLE RESPONSE
1. If 1/2 of a number is 4, how can I find the number?	1. Multiply 4 by 2.
2. If 3 times a number is 18, how can I find the number?	2. Divide 18 by 3. Take 1/3 of 18.
3. If 1/3 of my allowance is $1.00, how can I find the allowance?	3. Multiply $1.00 by 3.
4. If 4 times a number equals 40, how can I find the number?	4. Divide 40 by 4. Take 1/4 of 40.
5. If 1/2 of my friend's age is 7, how old is he?	5. Multiply 7 by 2.
6. If 1/3 of another friend's age is 5, how old is he?	6. Multiply 5 by 3.
7. If 2/3 of my sister's age is 10, how old is she?	7. Multiply 10 by 3/2. Divide 10 by 2/3.
8. If 1/5 of a number is 3, what is the number?	8. Multiply 3 by 5.
9. If 3/5 of a number is 15, how can I find the number?	9. Multiply 15 by 5/3. Divide 15 by 3/5.
10. If 5/6 of a piece of lumber is 15 feet long, how long is the entire piece?	10. Multiply 15 by 6/5. Divide 15 by 5/6.

Directions to Student:

You will be given a short verbal problem. You must tell what method of solution you would use. You are not to give the solution, but only the method to be used to find it.

Suggested Strategies:

After the class or group has gone over a number of such examples, try to get them to generalize. Ask them if they can find a relationship between the fractional part of the number given and the number they suggested as multiplier. They should find that

multiplication by the reciprocal of a number will undo multiplication by that number. Since the reciprocal is the multiplicative inverse, it undoes multiplication by that number.
If you wish to use division as the undoing operation for multiplication, this method may be used instead of reciprocals.

ACTIVITY 2. Win with Reciprocals.

Suggested Materials:
Each student receives a card on which are listed five open sentences, the solution of which requires use of reciprocals.

SAMPLE CARD:
1. $2/3\ n = 18$
2. $3n = 24$
3. $3/4\ n = 72$
4. $5/3\ n = 10$
5. $7n = 42$

Another group of cards, solution-cards, have written on them a number which will solve one of the sentences on a student-card. Each solution-card gives one number--the solution to one of the open sentences. These cards are shuffled and put on a desk in the front of the room or on the table around which a small group of students may be seated. The top card is turned over and the solution read. The students are given a minute or two to examine the sentences on their cards. If the solution fits one of his sentences, student raises his hand. Solution is checked. If correct, he receives a point.

Directions to Students:
You will receive a card on which are listed some open sentences. I have a pack of cards with the solutions to the sentences on your cards. If I turn up a number which solves one of your sentences, raise your hand. If correct, you will receive a point.

Suggested Strategies:
After two or three solution-cards have been turned up, student should exchange their sentence-cards since, by that time, they will have the solutions worked out. This activity can be used with any type of linear equation, not only those whose solution involves use of reciprocals.

COMPETENCY: Solve linear equations through use of the axioms.

OBJECTIVE: Student will be able to solve open sentences through use of the axioms.

ACTIVITY 1:

Suggested Materials:

1. Set of cards on which are listed open sentences, the solution of which requires use of one or more of the axioms. Each card should list one open sentence.
2. Set of cards on which are listed the symbols: X, ÷, +, - . Each card should list only one symbol. There should be more than one card listing each one of the four symbols.

Directions to Students:

You will receive five cards, each listing an open sentence. There are also cards on which are written the symbols: X, ÷, + and - . Each student will have a turn to pick one of these cards. If you can solve your open sentence with the symbol you picked, you must give the solution and explain how you used the symbol on the card to solve that particular open sentence. A correct use of the symbol earns you two points. After you correctly solve an open sentence place it and the symbol card you used to solve it on the desk in front of you. If you cannot solve any of your open sentences with the symbol card you picked, return it to the bottom of the pack. Play passes to the next student. If you need two symbol cards to solve one of your sentences and pick one which you can use, keep it in your hand until your turn comes around again. When you pick the second symbol required, play them both with the open sentence. This gives you three points. Play stops when a player has no more cards in his hand. Players count points played. Each card played, sentence cards and symbol cards, counts one point. You must deduct one point for each card remaining in your hand. Points are tabulated and play begins again, with each student receiving five cards. Student with highest number of points at end of game wins.

Suggested Strategies:

Activity can be used with the entire class or with a small group.

Instead of open sentences, verbal problems could be typed on cards. This would add to the difficulty since student would first need to write an open sentence to fit his verbal problem and then decide what operation he needs to use to solve it.

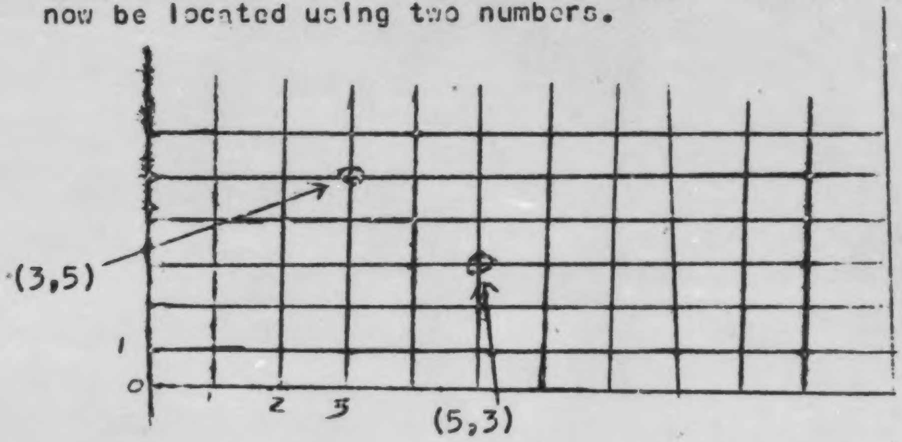
COMPETENCY: Construct a graph indicating the relationship of two variables from a given set of data.

OBJECTIVE: Given a grid with a horizontal scale and a vertical scale, the student should be able to locate on this grid points corresponding to an ordered pair.

ACTIVITY I ORDERED PAIRS ON A GEO-BOARD

Suggested materials: 1. peg board and golf tees 2. ordinary board with nails in place as in the diagram below 3. rubber bands

Directions for the teacher: Place one rubber band down the left column and one rubber band across the bottom row of the geo-board. Think of the rubber band enclosed nails as being numbered like the ones pictured. Any nail on a geo-board can now be located using two numbers.



Nail (3,5) is located by starting at nail (0,0).

1. The first number 3 tells how far to count OVER on the bottom row
2. The second number 5 tells how far to count UP from the bottom row.

The two numbers are called an ordered pair because the order in which the two numbers are written is important. For example, the nail located by the ordered pair (5,3) is not the same as the nail located by the ordered pair (3,5).

EXERCISES: Locate each of the following ordered pairs on the geo-board.

1. (4,2)
2. (9,9)
3. (0,6)
4. (5,3)
5. (1,7)
6. (0,8)

ACTIVITy 2

Directions: Use the seating chart below to answer the following questions.

row number						
6	Ben		Lem	Kay		Meg
5	Lynn	Don	Gail	Jane	Sue	Ed
4	AL	Sam	Pam	Greg	Gary	Ken
3	Ann	Abe	Eve	Matt	Ron	Leo
2	Bill	Peg	Mike	Paul	Dora	Liz
1	Dana	Jim	Rick	Lou	Pat	Jack
	1	2	3	4	5	6

column number

1. Who sits in the seat in column 4 row 3?____________
The ordered pair for this seat is (4,3). We call two numbers in a certain order an ordered pair.

ordered pair (4,3)
column row

2. Which student is at (5,2)?________________

3. Which student is at (3,5)?________________

4. Which student is at (5,3)?________________

5. Do (5,3) and (3,5) locate the same seat?____

6. Place a circle around the names of the students at these seats. (5,1), (5,2), (5,3), (5,4), (5,5). Are these students sitting in a straight line?____________.

7-10. In table I below, list the ordered pairs that locate the seats. If the seats form a straight line write "yes" in the last box. If the seats do not form a straight line write "no" in the last box.

Table I

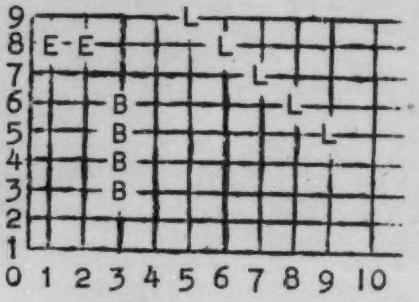
Classroom seats	ordered pairs	Straight line
column number is the same as the row number		
column number is 3 more than the row number		
sum of column number and row number is 7		
column number is less than row number		

ACTIVITY 3 GRID FOOTBALL

Object: To hit all of the other player's defensive men.

Rules: 1. Locate your 3 sets of defensive men on the defense grid. The men in each set are placed next to each other in vertical, horizontal, or diagonal straight lines like those below.

Kind of defense men	number of each kind	label for locations
Lineman	5	L
Back	4	B
End	2	E



Do not tell the other player where you placed your defensive men.

2. Player 1 takes 4 hits at the other player's defensive men by naming 4 ordered pairs on the grid. He records them on his offense grid. Player 2 records the hits player 1 has taken on his defense grid.

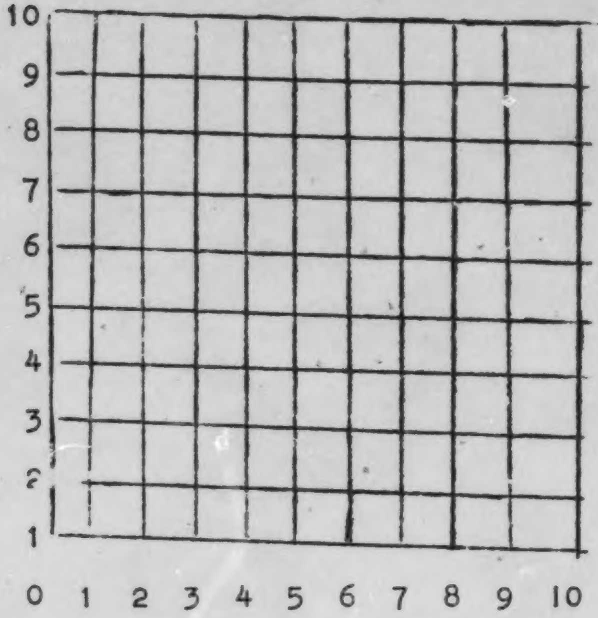
3. If player 1 has hit a defensive man, player 2 tells him the point and the kind of defensive man hit. Player 1 records this by placing a L, B, or E over that point.

4. After player 1 has taken 4 hits, player 2 does the same. Player 2 records the hits on his offensive grid, while player 1 records them on his defense grid.

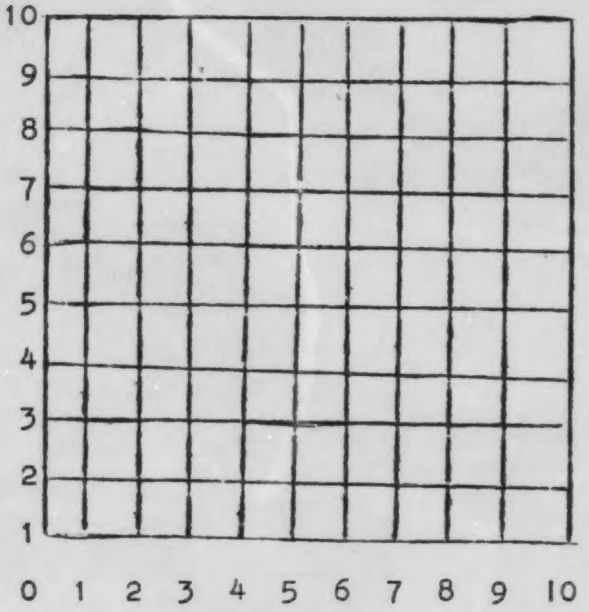
5. Players continue taking 4 hits at each turn until one has found all of the other's defensive men.

SCORING: The first player to hit all the other's defensive men wins.

OFFENSE GRID


10
9
8
7
6
5
4
3
2
1
0 1 2 3 4 5 6 7 8 9 10

DEFENSE GRID


10
9
8
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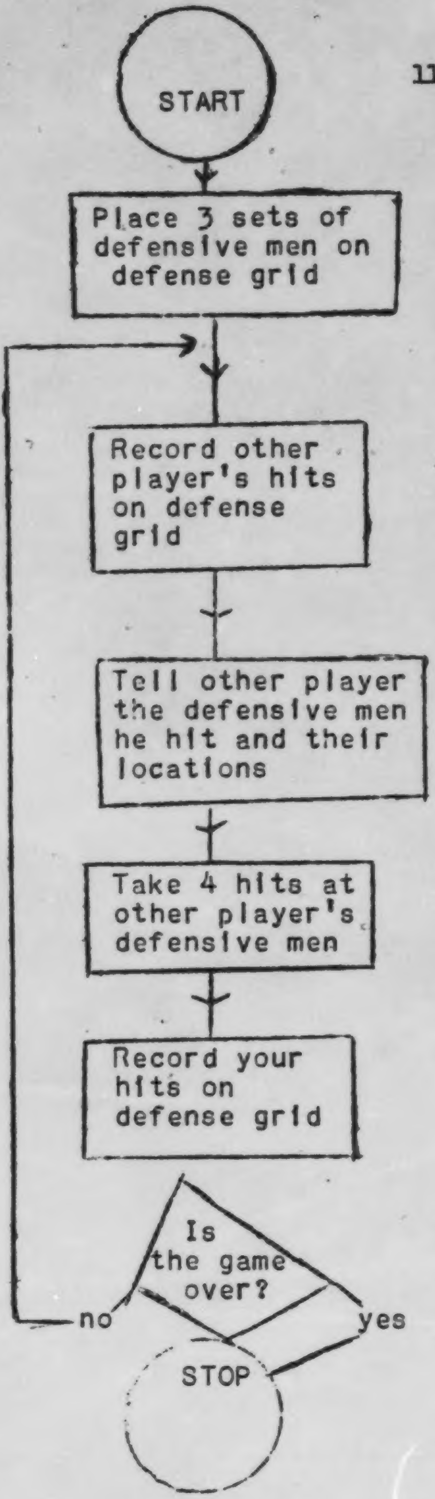
0 1 2 3 4 5 6 7 8 9 10


START
Place 3 sets of defensive men on defense grid
Record other player's hits on defense grid
Tell other player the defensive men he hit and their locations
Take 4 hits at other player's defensive men
Record your hits on defense grid
Is the game over?
no
yes
STOP

ACTIVITY 4 GAME: SPIN AND WIN

Rules:

1. One player uses X's. The other uses O's.
2. Use a pair of dice, one being red and the other white.
3. One player throws the dice.
4. The player then writes the ordered pair, (red, white) on his score card.
5. The player then places X or O (which ever he choose) on the point named by the ordered pair.
6. If this point is already taken, the player does not make a mark.
7. Each player takes 30 turns.
8. The player with the most points wins.

SCORING:

Three marks next to each other in a line gives 5 points.

Four marks next to each other gives 8 points.

Five marks next to each other in a line gives 16 points.

Six marks next to each other in a line gives 25 points.

SCORE CARD

1. (__,__)
2. (__,__)
3. (__,__)
4. (__,__)
5. (__,__)
6. (__,__)
7. (__,__)
8. (__,__)
9. (__,__)
10. (__,__)
11. (__,__)
12. (__,__)
13. (__,__)
14. (__,__)
15. (__,__)
16. (__,__)
17. (__,__)
18. (__,__)
19. (__,__)
20. (__,__)
21. (__,__)
22. (__,__)
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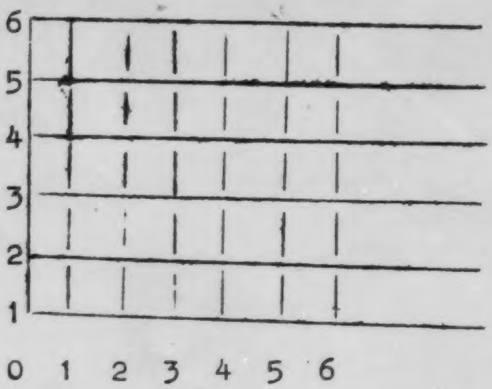
24. (__,__)
25. (__,__)
26. (__,__)
27. (__,__)
28. (__,__)
29. (__,__)
30. (__,__)



ACTIVITY 5 SECRET CODE

Directions: Use the secret code table to answer the following questions.

4	J	S	E	P	H
3	U	C	O	B	Y
2	T	L	F	X	N
1	G	R	V	D	W
0	A	M	Q	I	K
	0	1	2	3	4

1. Decode: (4,3) (2,3) (0,3) __ (0,0) (1,1) (2,4) __
(1,3) (1,2) (2,4) (2,1) (2,4) (1,1)
Hint: There are three words

2. Something has been done to make this next message harder to decode. You will have to think a little before you can spell out this message.

(3,2) (1,2) (-1,2) __ (0,3) (3,3) (-1,-1) (0,1) (0,1) __
(-1,0) (1,3) (-1,1) __ (-1,-1) (3,1) __ (-1,-1) __
(-1,1) (1,2) (2,0) (-1,-1) (3,2)

3. What letter is not in the table? Think of a pair of coordinates for this letter.

4. What letter is at the origin in the table?

5. Use the table to code your name.

ANSWERS

1. You are clever.

2. One has been subtracted from each coordinate.
Message: You shall get A for today.

3. Z

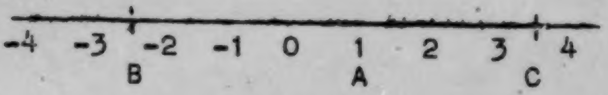
4. "A"

Suggestions to the teacher: On question 5, have the students code their name on a slip of paper. Gather them in a box, mix them and have students draw out the slips. Each student must try to figure out whose paper they have.

ACTIVITY 6

Directions for the student:

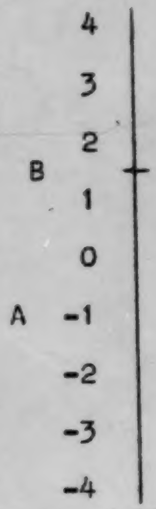
1. We named points of the number line by matching a number with a point.



2. Can we name points on a vertical number line in the same way?

   a. What is the point matching 3/2?

   b. What is the point matching -1?



3. How can we locate a point neither directly above or below zero nor directly at the right or left of zero? Use the graph below for the following questions.

   a. Is point A two units to the right and one up?

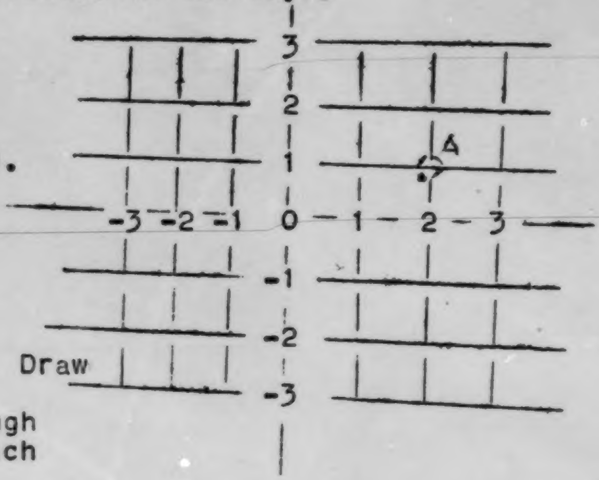
   b. Or is A one unit up and two to the right?

   c. Does either description locate point A equally well?

4. Where is point B? describe its location in two ways.

5. If we agree to give the number to the right or left first and then the number up or down, description of the location is simple. Point A is located at (2,1); B at (-3,-1).

6. Such number pairs are called ordered pairs, since we always give them in the same order.



EXERCISE: To be done by the student. Draw a vertical and horizontal number lines at right angles through point O. On your graph, locate each point listed below.

1. A (3,2)
2. C (4,-2)
3. B (-3,-1)
4. D (3/2,4)
5. E (-5,-3)
6. F (-2,-3)
7. G (7/2, 5/2)
8. H (6,-3)
9. (-1,5)

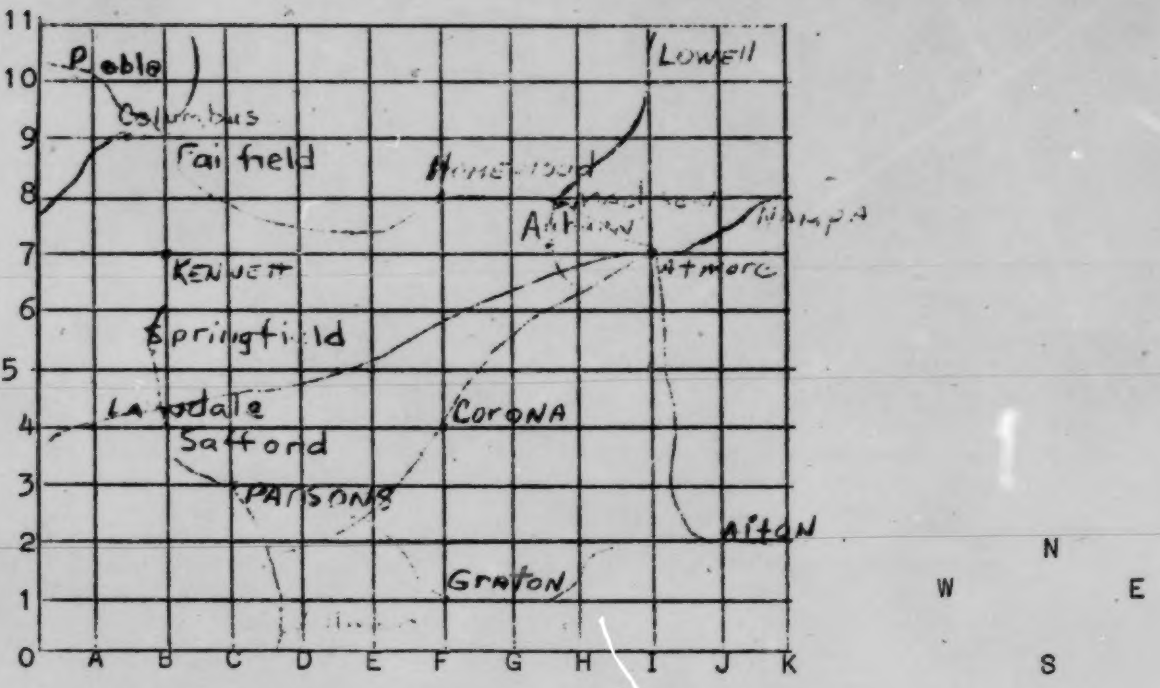
COMPETENCY: Construct a graph indicating the relationship of two variables from a given set of data.

OBJECTIVE: Given a point on a grid, the student should be able to write the ordered pair that describes the location of this point.

ACTIVITY 1

Suggested materials: Ditto maps, maps of your city or state.

Directions: The letters and the numbers on the state road map below help you locate towns on the map. For example, the town of Parsons is a (C,3). Use the map to answer the following questions.


Pueblo
Columbus
Fairfield
Homestead
Lowell
Ashton
Nampa
Kennett
Atmore
Springfield
Lansdale
Safford
Parsons
Corona
Alton
Graton
11
10
9
8
7
6
5
4
3
2
1
0
A
B
C
D
E
F
G
H
I
J
K
N
W
E
S

Use a letter and a number to tell where each town is located.

1. Fairfield________________ 3. Kennett________________

2. Graton________________ 4. Pueblo________________

Name the town located at each of the points below.

5. (H,10)________________ 7. (F,8)________________

6. (A,4)________________ 8. (F,4)________________

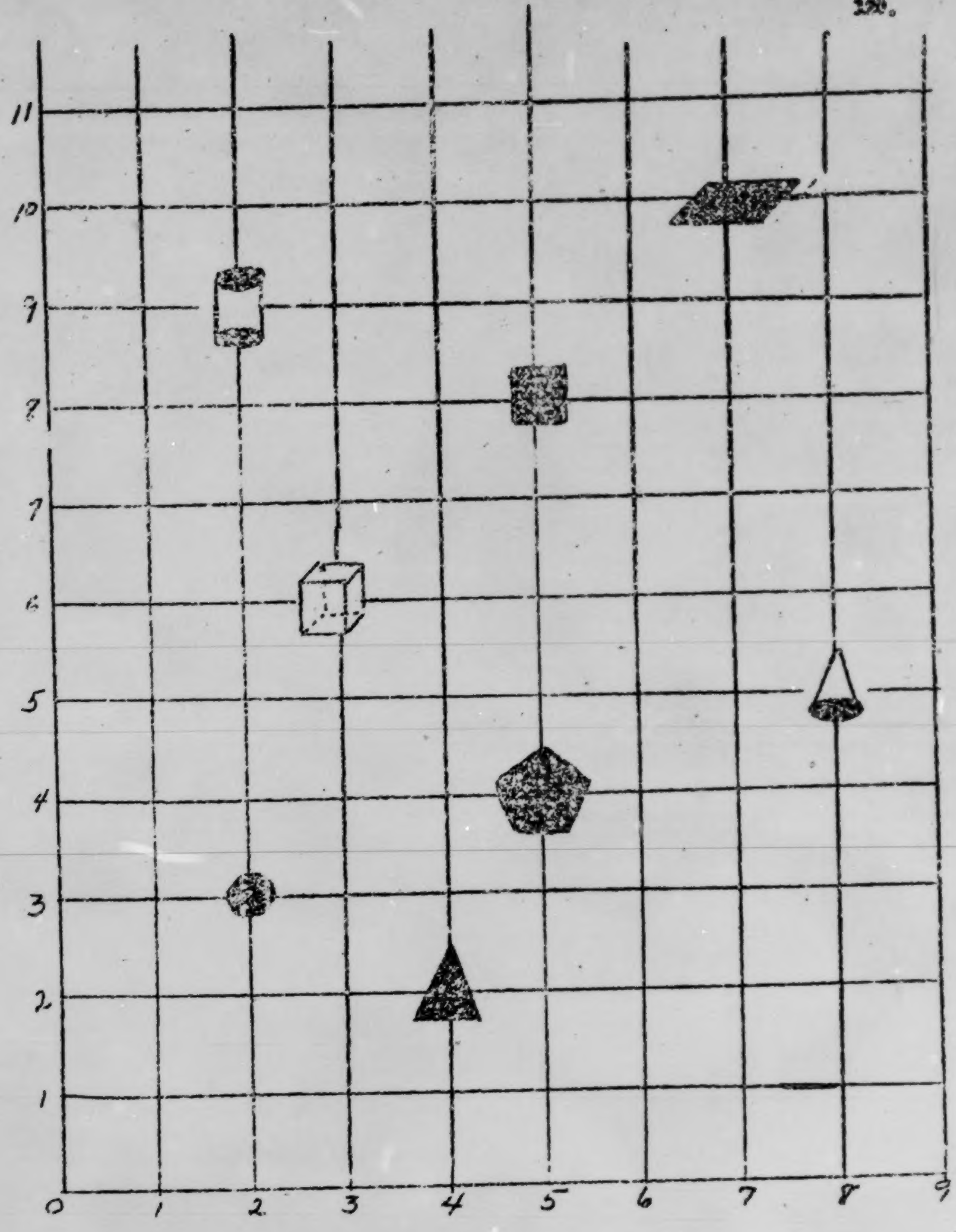
ACTIVITY 2 LOCATING POINTS

Suggested materials: An overhead visual like the one on the next page.

Directions for the teacher: Place the grid with geometric shapes on the overhead projector. Ask students what the ordered pairs are that locate the: a) cylinder b) cone c) cube d) square e) triangle f) circle g) parallelogram

Suggested strategy:

1. Students could direct this activity and call on others.
2. Individuals could be called on to give the ordered pairs.
3. The teacher or student could give the geometric figure and have a student give the ordered pair.
4. If the students are not able to recognize the geometric figures , the teacher could point out the objects.


11
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9

COMPETENCY: Construct a graph indicating the relationship of two variables from a given set of data.

OBJECTIVE: Given a grid and a set of ordered pairs the student should be able to draw the design formed by the points located by the ordered pairs.

ACTIVITY 1

Suggested materials:
different size graph paper

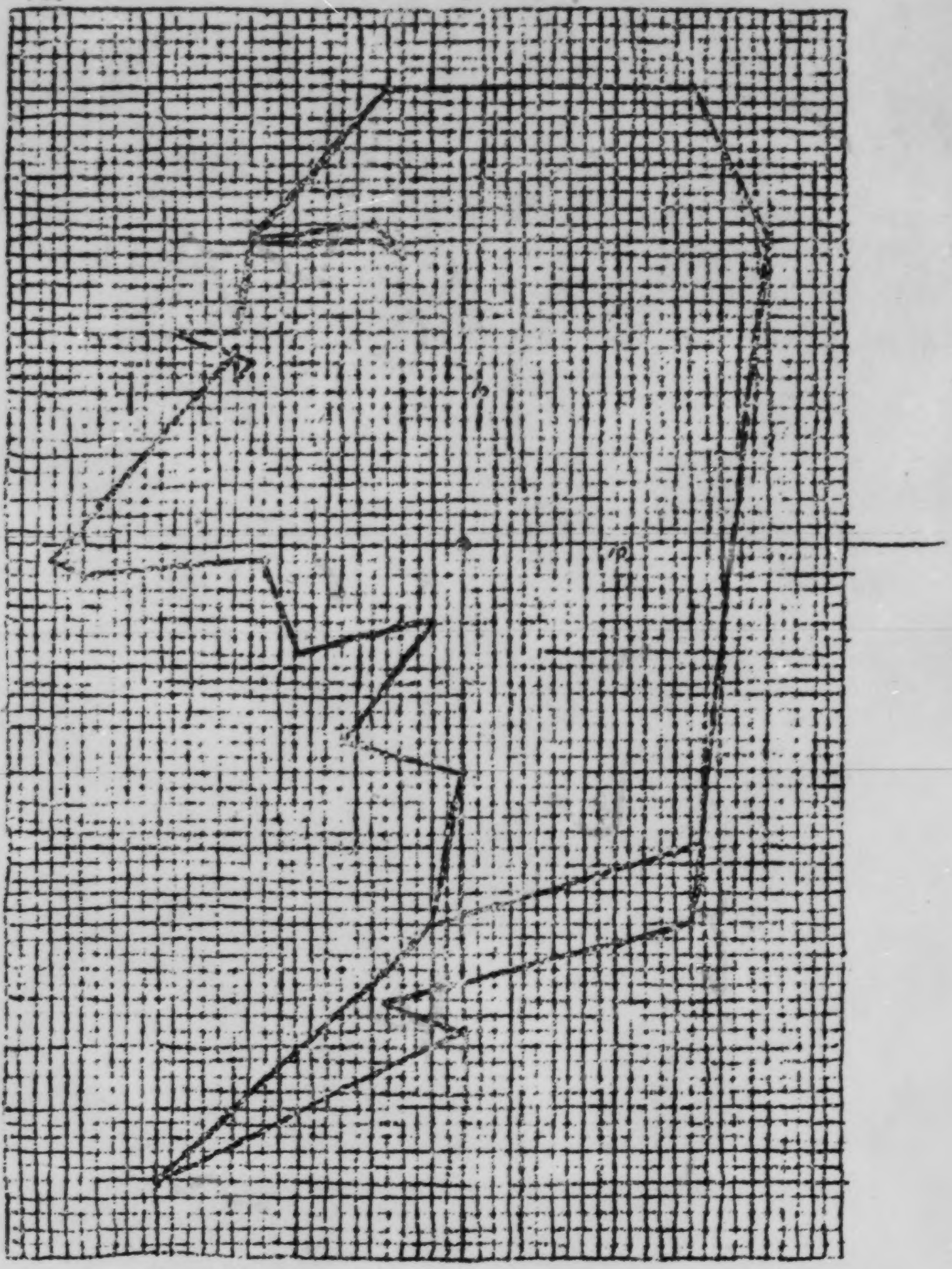
Directions to student:
Plot the points in the order given. Connect with straight lines point "a" with point "b", point "b" with point "c", etc. until the last two points given are connected. Follow this procedure for the next four graphs.

Suggested strategy: Have students do 1, 2, 3, and/ or 4 of these graphs.

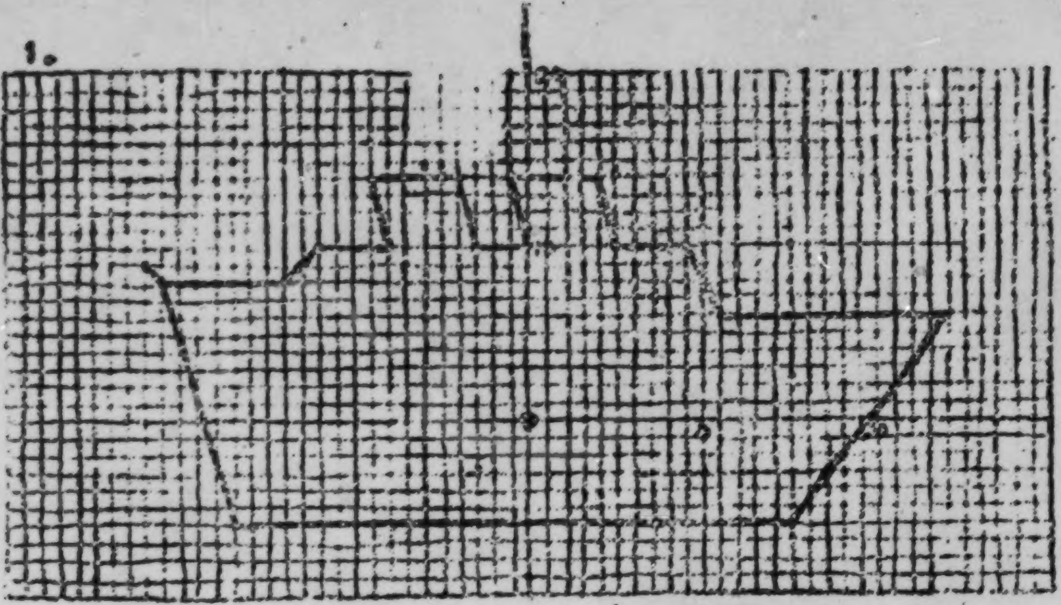
1. a.	(-22,9)	2. a.	(-3 1/2, -10)	3. a.	(15,-20)	4. a.	(-14,14)
b.	(-21,8)	b.	(-3 1/2, -15)	b.	(-2,-25)	b.	(-15,11)
c.	(-17,-6)	c.	(-5 2/3, -17)	c.	(-20,-42)	c.	(-17,10)
d.	(15,-6)	d.	(-5 2/3, -12)	d.	(0,-32)	d.	(-11,8)
e.	(24, 6)	e.	(-3 1/2, -10)	e.	(-5,-30)	e.	(2,13)
f.	(11, 6)	f.	(-3 1/2, -6)	f.	(15,-25)	f.	(17,8)
g.	(9, 10)	g.	(-2 3/4, -5)	g.	(20,20)	g.	(26,8)
h.	(5, 10)	h.	(-2 1/2, 2 1/2)	h.	(15, 30)	h.	(37,15)
i.	(4, 14)	i.	(-2, 3 1/2)	i.	(-5,30)	i.	(30,6)
j.	(-1, 14)	j.	(-1 1/2, 12 1/2)	j.	(-14,20)	j.	(12,4)
k.	(0,10)	k.	(0, 14 1/3)	k.	(-6,21)	k.	(1,-4)
l.	(-3, 10)	l.	(1 1/2, 12 1/2)	l.	(-4, 19)	l.	(0,-10)
m.	(-4, 14)	m.	(2, 3 1/2)	m.	(-6,20)	m.	(-11,-17)
n.	(-9, 14)	n.	(2 1/2, 2 1/2)	n.	(-14,20)	n.	(-11,-15)
o.	(-8, 10)	o.	(2 3/4, -5)	o.	(-15,14)	O.	(-2,-9)
p.	(-12, 10)	p.	(3 1/2, -6)	p.	(-19,14)	p.	(-1,0)
q.	(-14, 8)	q.	(3 1/2, -15)	q.	(-14,12)	q.	(-5,0)
r.	(-21, 8)	r.	(5 2/3, -17)	r.	(-15,11)	r.	(-5,-4)
		s.	(5 2/3, -12)	s.	(-15,13)	s.	(-4,-7)
		t.	(3 1/2, -10)	t.	(-27,-1)	t.	(-6,-5)
		u.	(3 1/2, -14)	u.	(-25,-2)	u.	(-7,0)
		v.	(-3 1/2, -14)	v.	(-13,-1)	v.	(-10,0)
				w.	(-11,-7)	w.	(-18,6)
				x.	(-2,-5)	x.	(-21, 4)
				y.	(-8,-13)	y.	(-22,5)
				z.	(0,-15)	z.	(-20,9)
				aa.	(-2,-25)	aa.	(-14,14)

The flag is missing from the pole (see -22,9 above). Design your own flag and give coordinates of its points.

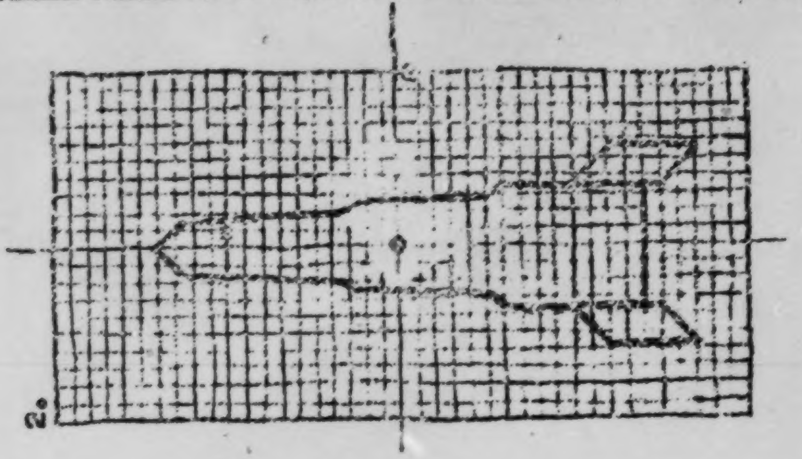
3/



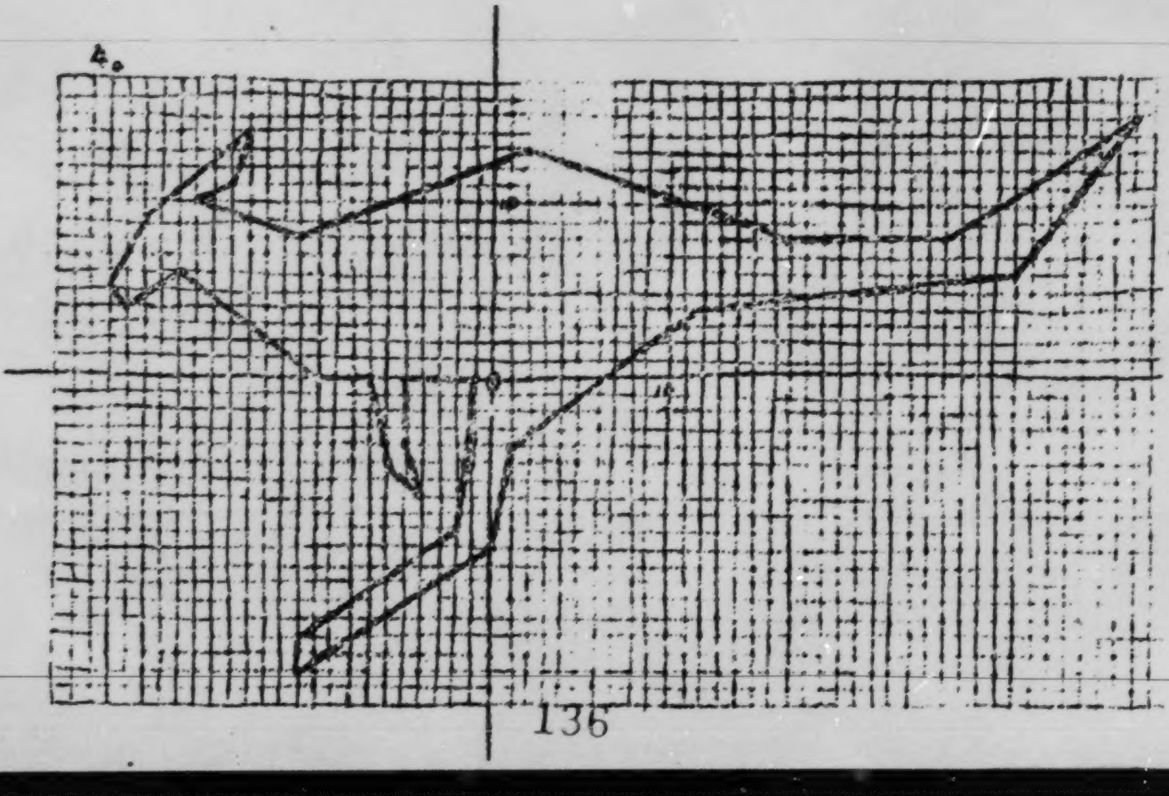
1.



2.



4.



ACTIVITY 2

Suggested material: Graph paper

Directions to student:

Plot the points in the order given. Connect with straight lines point "a" with point "b", point "b" with point "c" , etc. unless there is a directive to stop. A directive to stop means you do not connect these two points only, but all others are connected in order. Follow this procedure for the next 3 graphs.

Suggested strategy:

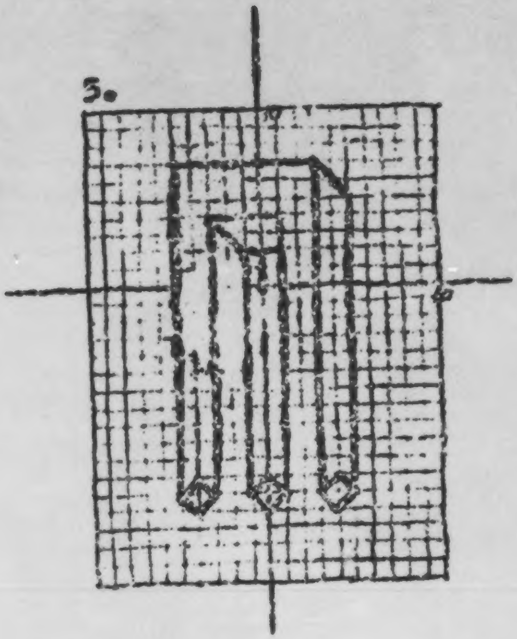
The teacher can have the students do 1, 2, and/or all 3 of these graphs.

5. a. (-5, -12)
b. (-4, -13)
c. (-3, -12)
d. (-3, 4)
e. (1, 4)
f. (1,2)
g. (-1, 2)
h. (-3, 4)
STOP
i. (1, 2)
j. (1, -12)
STOP
k. (-1, 2)
l. (-1, -12)
m. (0, -13)
n. (1, -12)
o. (0, -11)
p. (-1, -12)
STOP
q. (3, 7)
r. (5,5)
s. (5, -12)
t. (4, -13)
u. (3, -12)
v. (4, -11)
w. (5, -12)
STOP
x. (-3, -12)
y. (-4, -11)
z. (-5, -12)
aa. (-5, 7)
bb. (3,7)
cc. (3, -12)

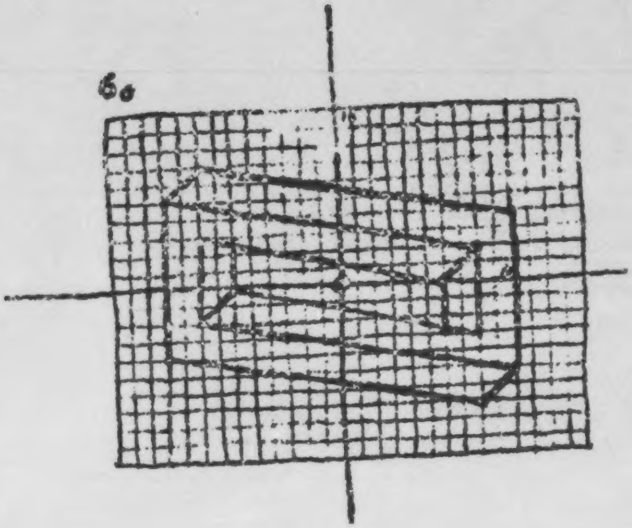
6. a. (8,2)
b. (6,0)
c. (-8,3)
d. (-8, -2)
e. (10, -5)
f. (8, -7)
g. (-10, -4)
h. (-10, 5)
i. (8, 2)
j. (8, -3)
k. (-6,0)
l. (-6, 2 1/2)
STOP
m. (-6, 0)
n. (-8, -2)
STOP
o. (6, 0)
p. (6, -2 1/2)
STOP
q. (-10, 5)
r. (-8,7)
s. (10, 4)
t. (10, -5)
STOP

7. a. (1, 11 1/3)
b. (3 1/2, 11)
c. (4, 10)
d. (3 2/3, 9 2/3)
e. (2 1/2, 10)
f. -1/2, 10)
g. (-3/4, 12)
h. (-1,12 1/2)
i. (-1 1/2, 14 1/2)
j. (-2 1/3, 15)
k. (-2, 15 1/2)
l. (-3, 15 1/3)
m. (-2 2/3, 15)
n. (-3, 15)
o. (-4, 14)
p. (-3 1/2, 12 1/2)
q. (-3, 12)
STOP
r. (8, 10)
s. (9 1/2, 10)
t. (11 1/2, -1/2)
u. (8 1/2, -1)
v. (8 1/2, -7)
w. (-6 1/2, -7)
x. (-6 1/2, -1)
y. (-9 1/2, -1/2)
z. (-7 1/2, 10)
aa. (-3 1/2, 10)
bb. (-4 1/3, 11)
cc. (-3 1/2, 12 1/2)
STOP
dd. (4, 10)
ee. (8, 10)
ff. (7 3/4, 12 3/4)
gg. (6 3/4, 13 1/2)
hh. (6, 12 2/3)
ii. (6 3/4, 11)
jj. (5 1/3, 11)
kk. (4 2/3, 11 3/4)
ll. (3 2/3, 13)
mm. (2 1/3, 13 2/3)
nn. (1 1/2, 13 1/2)
oo. (0,12)
pp. (-3/4, 12
qq(-1/2,10)
rr. (-1 3/4, 10)
ss. (-2 1/3, 7 1/2)
tt. (-2 1/2, 7)
uu. (-3, 7 1/2)
vv. (-3,10)
ww. (-3 1/2, 10)

5.



6.



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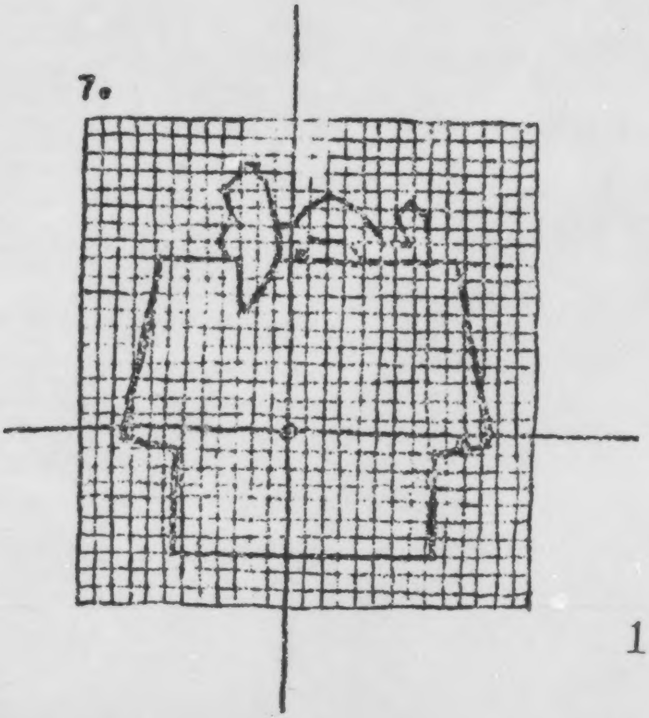


COMPETENCY: Describe parallel lines, perpendicular lines and intersecting lines using drawings or intuitive concepts.

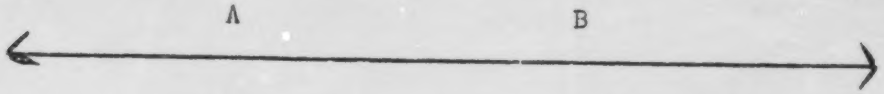
OBJECTIVE: The student can recognize a line segment from drawings and objects in his surroundings.

ACTIVITY 1:

Suggested materials:
Geoboard, rubber bands, straight edge and pencil, copy of student directions.

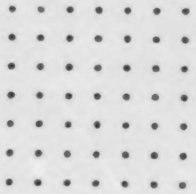
Directions to student:

1. Hook a rubber band around any two nails on the geoboard. The rubber band now represents a line segment. The nails represent points. A line segment begins and ends at two points. A line segment is part of a line. Look at the example.



Capital letters are used to mark points. The part of the line between point A and point B is the line segment AB. The arrows show that the line has no end points and is straight.
Practice making line segments on the geoboard.

2. On the geo board show four line segments. With straight edge and pencil, show the four line segments you have on the geoboard on the picture of the geoboard below.



3. Label the four line segments in the picture above as line segments AB; CD; EF; and MN.

4. Give five examples of line segments in this room.

5. When you have finished this activity, report to your teacher.

Suggested strategies:

Use individually or in groups of two.
If this is the first time students are exposed to the geoboard, an explanation will be needed to establish the idea that the nails represent points in a plane and the rubber bands represent line segments. Allow a little time for them to experiment.
Each student should have a copy of the directions to the student.
If students are working individually each student needs a geoboard. If working in groups each group needs a geoboard.

ACTIVITY 2:

Suggested materials:

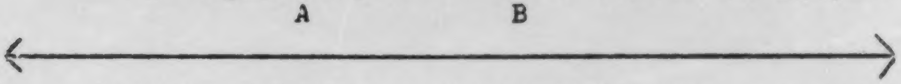
A sheet of unlined paper and pencil, copy of student directions.

Directions to Student:

1. Take the sheet of unlined paper. Start in one corner and letter the corners in a clockwise direction A, B, C, and D.

2. Fold the paper so that the corner lettered C touches the side AD.

3. Unfold the paper. The crease that you made represents a line segment.
   A line segment is part of a line. Look at the example.



   Capital letters are used to identify points. The part of the line between the point A and the point B is the line segment AB.
   The arrows indicate that the line has no end and is straight. Label the line segment you made MN.

4. Fold the paper and make two more line segments. Identify these line segments using letters not already used.

5. There are other examples of line segments on this paper. Title the paper "Line Segments," and list at least six line segments.
   Example: line segment MN.

6. Describe five examples of line segments in this room.

7. You have finished this activity. Report to your teacher.

Suggested Strategies:

Use individually or in groups of two.

COMPETENCY: Describe parallel lines, perpendicular lines and intersecting lines using drawings or intuitive concepts.

OBJECTIVE: The student can recognize and describe parallel lines from drawings and objects in his surroundings.

ACTIVITY 1:

Suggested materials:
Geoboard, rubber bands, straight edge, and pencil, copy of student directions, worksheet A

Directions to student:

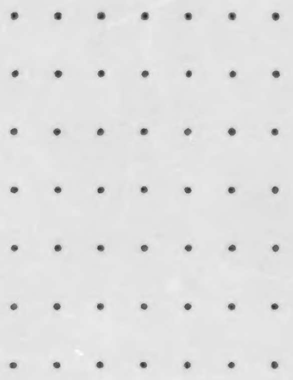
1. If you hook a rubber band around any two nails on the geoboard, the rubber band will represent a line segment. Practice representing several line segments in this way. Clear the geoboard of rubber bands.

2. On the geoboard make a line segment by hooking a rubber band around the second and the last nails in the second row of nails. Leave this rubber band on the board.

3. Make another line segment by hooking another rubber band around the first and sixth nails in the third row. With these two line segments you have an example of parallel line segments. Line segments are parallel if they are always the same distance apart and these are the same distance apart. Leave these on the board.

4. Make two more pairs of parallel line segments on the geoboard.

5. Use the straight edge and pencil and show the three pairs of parallel lines you have on the geoboard on the picture of the geoboard.



6. Describe five examples of parallel line segments in this room and tell why they are parallel.

7. On the worksheet you will see pictures of line segments. Decide which pictures show parallel line segments, and write "parallel lines" in the blank following the number of the picture. Tell why they are parallel.

8. You have finished this activity. Report to your teacher.

Suggested Strategies:

If this is the first time students are exposed to the geoboard an explanation will be needed to establish the idea that the nails represent points in a plane and the rubber band represents line segments. Allow a little time for them to experiment.
Use individually or in small groups.
If students are working individually each student needs a geoboard. If in small groups each group will need a geoboard.

ACTIVITY 2:

Suggested materials:

A sheet of unlined paper approximately 8" x 11" and pencil; copy of Worksheet A; copy of student directions.

Directions to student:

1. Take the sheet of unlined paper, start in one corner and letter the corners in a clockwise direction A, B, C, and D.

2. Fold the paper so corner D falls on corner A and corner C falls on corner B.

3. Unfold the paper. This crease represents a line segment. Use the letters EF at the ends so as to identify it.

4. Now fold the paper so the line segment D C falls on the line segment E F.

5. Unfold the paper. Use the letters G H to identify this crease as a line segment. These two creases or line segments E F and G H represent parallel lines. Parallel lines are always the same distance apart.

6. There are other examples on this paper of parallel line segments. Title this paper "Parallel Line Segments." State what line segments are parallel.
   Example: line segment A B is parallel to line segment G H.

7. Describe five examples of parallel lines in this room and tell why they are parallel.

8. On the workshhet A you will see pictures of line segments. Decide which pictures show parallel line segments and write "parallel lines" in the blank following the number of the picture and tell why they are parallel.

9. You have finished this activity, report to your teacher.

Suggested Strategies:
Use individually or in small groups.

COMPETENCY: Describe parallel lines, perpendicular lines and intersecting lines using drawings or intuitive concepts.

OBJECTIVE: The student will be able to recognize intersecting lines.

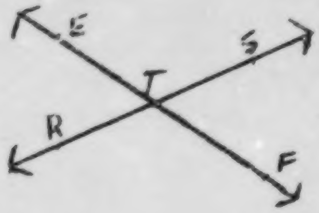
ACTIVITY 1:

Suggested materials:
unlined paper and pencil, copy of worksheet A, copy of student directions

Directions to student:

1. Take a sheet of paper and letter the four corners A, B, C, and D in a clockwise direction.
2. Fold the paper so that A falls on C. Letter this crease E F.
3. Now fold so that B falls on D. Letter this crease R S. These two creases, E F and R S represent intersecting line segments. If they continued on and on, they would represent intersecting lines. Intersecting lines mean that they cut across or meet one another at one point (which is their only common point) and we call this point the point of intersection.

   Illustration:

   Written in symbols:

   $\overleftrightarrow{EF} \cap \overleftrightarrow{RS}$ is point T.

4. Fold three more examples of intersecting lines and letter them. Then write what happened using symbols.
5. Describe 5 examples of intersecting line segments in this room.
6. On Worksheet A you will find pictures of line segments. Decide which represent intersecting line segments and write it in the blank following the number.

Suggested Strategies:
Students will need to work either individually or in pairs. If the work on perpendicular lines has preceded intersecting lines, it should be called to the attention of the student that they are formed by a unique intersection.

ACTIVITY 2:

Suggested materials:
Geoboard, rubber bands, straight edge, pencil

Directions to student:

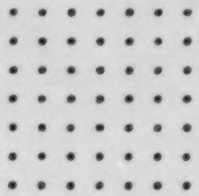
1. On the geoboard represent a line segment by hooking the rubber band around nail 1 row 4 and nail 4 row 1.

2. Hook another band around nail 1 row 1 and mail 2 row 6. If each band continued, it would represent a line. These are intersecting lines meaning that they cut one another at a point.

3. Hook a band around nail 6 row 2 and nail 6 row 6.

4. Hook a band around nail 4 row 4 and another around nail 7 row 4. Are these intersecting line segments? Do you notice any difference in the way these two sets intersect?

5. Use the straight edge and pencil to show the intersecting line segments on your geoboard using these dots as the nails.



6. Clear your geoboard. Represent 2 more pairs of intersecting line segments on it.

7. Describe 5 examples of intersecting line segments in this room.

8. On worksheet A you will find pictures of line segments. Decide which represent intersecting line segments and write it in the blank following the number.

Suggested Strategies:
If this is the first time students have used the geoboard, an explanation will be needed to establish the idea that nails represent points in a plane (the board).
It is better for each student to have a geoboard but if that many are not available, they can work in groups as small as possible.

COMPETENCY: Describe parallel lines, perpendicular lines and intersecting lines using drawings or intuitive concepts.

OBJECTIVE: The student will be able to recognize perpendicular lines.

ACTIVITY 1:

Suggested materials:
unlined paper, pencil, copy of student directions, workshhet A

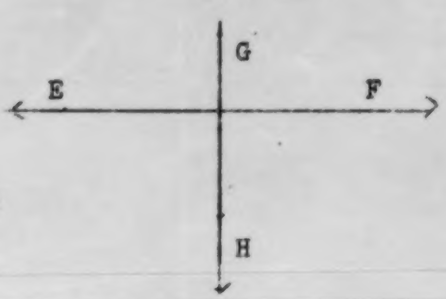
Directions to student:

1. Take a sheet of paper and letter the four corners A, B, C, and D in a clockwise direction.

2. Fold the paper so that D falls on A and C falls on B. Letter this crease E F.

3. Then fold so that A falls on B and D on C. Unfold and letter this crease G H. These two creases E F and G H represent perpendicular line segments. If they continued on and on, they would represent perpendicular lines. Notice that they intersect in such a way that square corners are formed. We call these right angles. When right angles are formed, we have perpendicular lines. We say one line is perpendicular to another.

   Illustrated as follows:



   Written in symbols as follows:

   $\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$ or $\overleftrightarrow{GH} \perp \overleftrightarrow{EF}$

4. Turn your paper to several positions. Did E F remain perpendicular to G H ?

5. Using another sheet of paper, fold an example of perpendicular lines, naming them line A B and line C D.

6. Describe five examples of perpendicular line segments in this room.

7. On worksheet A you will find pictures of line segments. Decide which represent perpendicular line segments and write it in the blank following the number.

Suggested Strategies:
Students will need to work individually or in pairs. The number of illustrations needed so that each student can recognize perpendicular lines may be varied according to the group.

ACTIVITY 2.

Suggested materials:
Geoboard, rubber bands, pencil, straight edge

Directions to student:

1. By hooking a rubber band around any two nails on the geoboard, the rubber band represents a line segment. Practice representing several line segments in this way and then clear the geoboard.

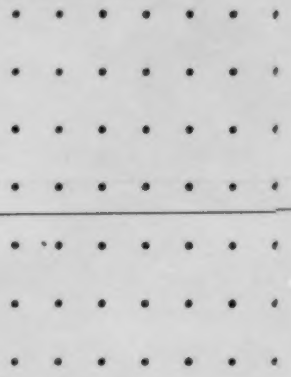
2. On the geoboard make a line segment by hooking a rubber band around nail 2 row 1 and nail 2 row 7.

3. Hook another band around nail 1 row 6 and nail 6 row 6. If each band continued, it would represent a line. These are perpendicular lines. They make square angles which are called right angles. Lines which meet at right angles are called perpendicular lines.

4. Now hook a band around nail 4 row 1 and nail 7 row 4.

5. Hook another band around nail 7 row 1 and nail 3 row 5. Do these represent perpendicular lines?

6. Use the straight edge and pencil to show the two pairs of perpendicular lines you have on your geoboard using these dots to represent the nails.



7. Clear your geoboard. Represent two more pairs of perpendicular line segments on it.

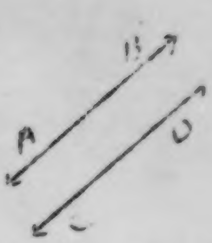
8. Describe five examples of perpendicular line segments in this room.

9. On worksheet A you will find pictures of line segments. Decide which represent perpendicular line segments and write it in the blank following the number.

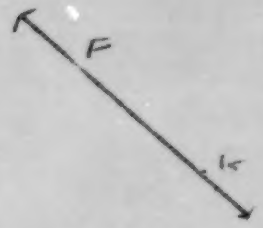
Suggested Strategies:
If this is the first time students have used a geoboard, an explanation will be needed to establish the idea that nails represent points in a plane. The number of nails used is immaterial as far as the understanding of perpendicular lines is concerned. Each student should have a geoboard but if that many are not available, they can work in groups.

Worksheet A

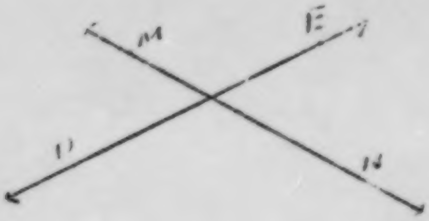
1. ______________________



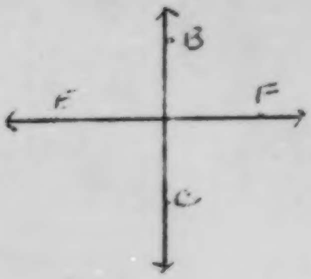
2. ______________________



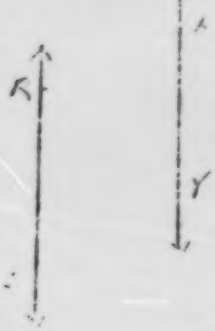
3. ______________________



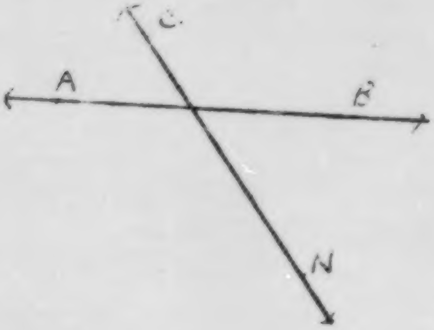
4. ______________________



5. ______________________

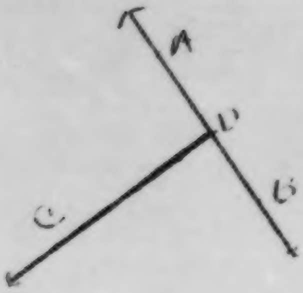


6. ______________________

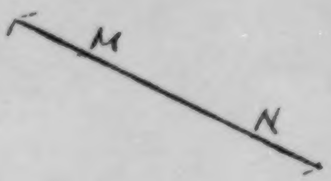


Worksheet B

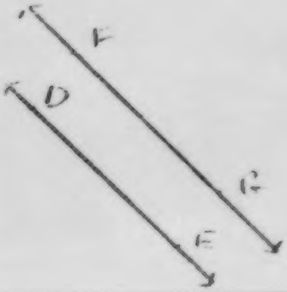
7. ______________________



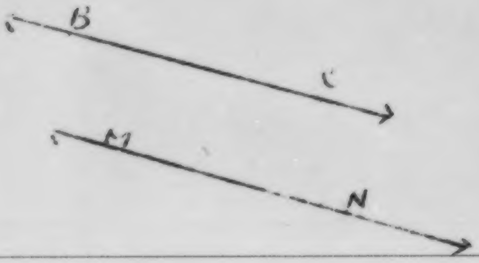
8. ______________________



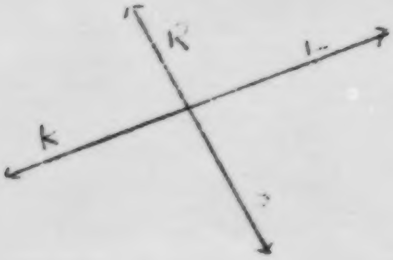
9 ______________________



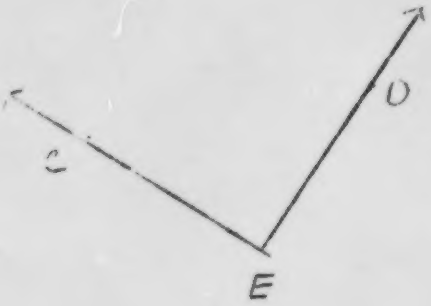
10 ______________________



11 ______________________



12 ______________________



COMPETENCY: Describe parallel lines, perpendicular lines and intersecting lines using drawings or intuitive concepts.

OBJECTIVE: To demonstrate the definition of parallel lines, perpendicular lines and intersecting lines by drawings, paper folding and observable relationships.

ACTIVITY 1: Demonstrating parallel lines by paper folding, and the use of a ruler and pencil and paper.

Suggested materials:

1. foot ruler
2. yard stick
3. lined paper

Directions to student:

1. Measure the distance between the top and bottom of the chalkboard at each end and record these distances. Are they different? If so, how much difference?
2. Measure the width of the top and the bottom of the door and record these measurements. Are they different? If so, how much difference?
3. Visualize railroad tracks. Do you think they are the same distance apart in town as they are a mile out of town? Explaine why or why not.
4. Observe the printed lines on your paper. If they were extended both directions would they meet?
5. Lay your foot ruler on your paper and without moving the ruler, draw lines along both edges. Will these lines meet if they were extended indefinitely in both directions.
6. The answers to parts "4" and "5" should have been NO. It means that the lines are PARALLEL to each other. PARALLEL means they would never meet no matter how far they were extended in either direction.
7. From the right side of your paper measure in 2 inches at the top and the bottom and then draw a line between these two points.
8. Follow the same procedure as in part "7" except now measure in 3 inches instead of 2 inches.
9. Are these two lines parallel to each other? How far are they apart at all points along the lines? Are these lines parallel to the edge of the paper?

Suggested strategies: (for teacher)

1. A teacher may demonstrate parallel lines by folding a sheet of paper in half and then folding again in the same direction. When the paper is opened the creases will represent parallel lines. You might want to teach the symbol (//) for parallel lines at this point.
2. Have available models of plane geometric figures with each vertice designated by a letter and have students determine how many pairs of parallel sides each has.
3. Make use of observable examples of parallel lines in the classroom.

ACTIVITY 2: Demonstrating intersecting lines by paper folding, and the use of ruler and paper and pencil.

Suggested materials:
1. foot ruler
2. yard stick
3. lined paper

Directions to student:
1. Using your ruler draw two lines on your paper that cross each other. These are called INTERSECTING lines. Do these lines intersect any of the printed lines on your paper?
2. Do you think the intersecting lines you drew in part "1" will intersect at any other point if extended?

Suggested strategies: (for teacher)
1. A teacher may demonstrate intersecting lines by folding a sheet of paper in two intersecting creases. It is suggested that the creases not be at right angles to each other for this part.
2. Have students draw 3, 4, or 5 lines that are all parallel to each other and another line that intersects each of them.
3. Make use of observable examples of intersecting lines in the classroom.

ACTIVITY 3: Demonstrating perpendicular lines by paper folding, and the use of a ruler, pencil and paper.

Suggested materials:
1. foot ruler
2. yard stick
3. lined paper

Directions to student:
1. Draw a line parallel to the top of your paper and another line parallel to the left side of your paper. Are these lines parallel to each other?
2. Place the corner of another sheet of paper at the intersection of these lines. Does the corner of the paper fit into any of the angles that are formed by the intersecting lines? Your answer should be YES. When this condition is true, then the two lines are perpendicular to each other. It is also said that the angles formed are called right angles and their measures are each 90 degrees.

Suggested strategies: (for teacher)
1. A teacher may demonstrate perpendicularity by folding a sheet of paper in half and folding it in half again so that the crease lies along itself. You might want to teach the symbol ($\perp$) of perpendicularity at this time.
2. Make use of observable examples of perpendicular lines in the classroom.

COMPETENCY: Classify simple plane figures by distinguishing some of their properties.

OBJECTIVE:

The student is able to group the triangles from a given set of triangles into subsets of equilateral, isosceles, and scalene triangles; and he is able to name the respective subsets.

ACTIVITY 1: A set of cardboard models of triangles is used to make cut-outs which are used for measurment of sides and are identified according to length of sides.

Suggested materials:

An assortment of cardboard or plastic models of triangles--at least two each of isosceles, equilateral, and scalene. Construct them in such a way that the sides are readily measured to the nearest inch. Label the models with different capital letters and each side of a triangle with a small letter.
Scissors, unlined paper, pencil, ruler.

Directions to student:

1. Trace around each of the models of triangles on your paper.
2. Label the triangles on your paper exactly like the cardboard models.
3. Cut out each of the triangles.
4. With your ruler measure each side of the triangles in whole number of inches. Check each measurement.
5. Write the number of inches next to the letters for the sides of each triangle.
6. Do you have a triangle with all three sides the same? If you do, write the name "EQUILATERAL" on it.
7. Do you have a triangle with exactly two sides the same? If you do, write the name "ISOSCELES" on it.
8. On the triangles that have all the sides a different length, write the name "SCALENE."
9. You should have all the triangles named. Study and learn the three names which tell how many equal sides each triangle contains.

Suggested strategies:

1. Review measurement to the nearest inch if necessary.
2. Ask students to pronounce names correctly. If they do not know, give them the correct pronunciation.
3. Have the students keep models for future reference. They can be used for angle relationships.

ACTIVITY 2: Drawings of a set of triangles are named with respect to length of sides.

Suggested matherials:
Worksheet W, pencil, ruler

Directions to student:

1. On Worksheet W, measure the length of each side of triangle A with your ruler. Check each measurement.
2. Write the number of inches for each side near the center of the side. Each number should be one of the set:

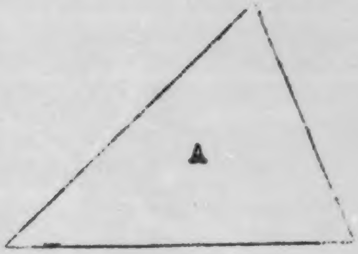
   $\{1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4, 4\frac{1}{2}, 5\}$

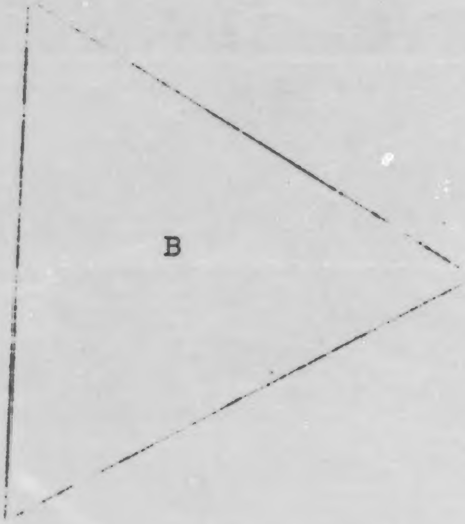
3. Do the same thing with each of the other triangles, B through G.
4. Do any of the triangles have all three sides the same length? If they do, write the word, "EQUILATERAL" on them.
5. Do any of the triangles have only two sides the same length? If they do, write the word "ISOSCELES" on them.
6. On the triangles that have three sides with different lengths write the word "SCALENE."
7. Study and learn the names which tell how many equal sides each triangle contains.

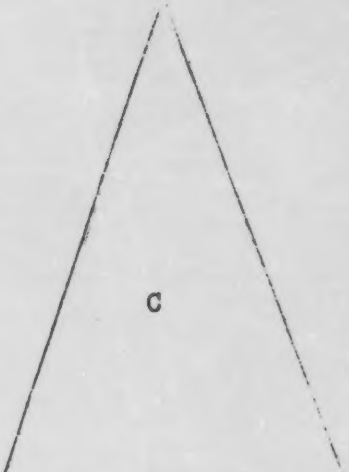
Suggested strategies:

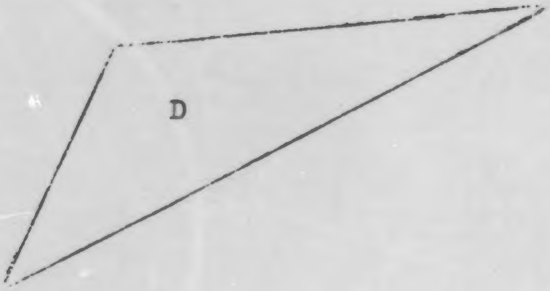
1. Review measurement to the nearest half-inch if necessary.
2. Have students practice correct pronunciation of the names of the triangles. Give them pronunciations.
3. If students have access to geoboard, have them illustrate types of triangles on it if they can.

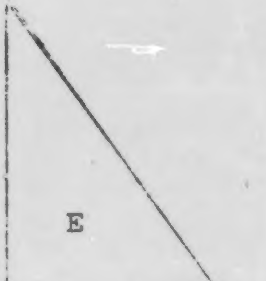
WORKSHEET W

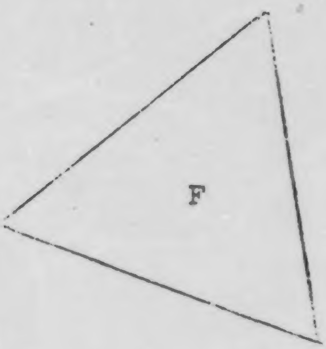


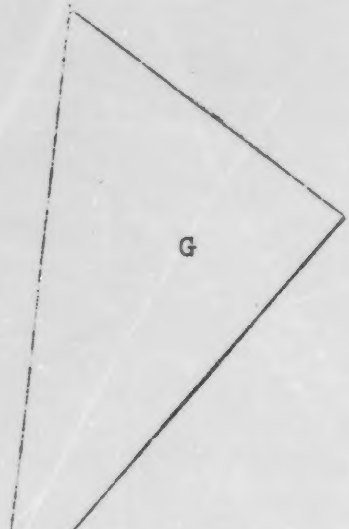












COMPETENCY: Classify simple plane figures by distinguishing some of their properties.

OBJECTIVES: The student is able to select the triangles and quadrilaterals from a set of models of polygons.

ACTIVITY 1: Selecting the set of all triangles and the set of all quadrilaterals from a set of cardboard models of polygons, and becoming familiar with names "triangle" and "quadrilateral."

Suggested materials:

paper, pencil, and cardboard or plastic models of polygons including scalene, isosceles, equilateral, acute, obtuse, and right triangles; quadrilaterals including square, rectangle, rhombus, parallelogram, trapezoid, and several with no congruent sides; and an assortment of other polygons. A label (capital letter) is written on each of the models.

Directions to student:

a. Select all the models with exactly three sides, and put them into one group.
b. On your paper next to the number 3, write the letter of the models with three sides.
c. Select all the models with exactly four sides, and put them into a single group.
d. On your paper next to the number 4, write the letter of the models with four sides.
e. If you think you know the name given to all figures with three sides, write it next to them on your paper.
f. If you think you know the name given to all figures with four sides, write it next to them on your paper.
g. How many three sided models did you find?
h. How many four sided models did you find?

Suggested strategies:

a. Students are likely to know the name "triangle." The word "quadrilateral" is likely to be new. After their attempts at naming, such as "square," "box," "rectangle," etc., have them try to find the name on their own--or use the "tell" method.
b. Indicate to students such names as "square," "rectangle" are special quadrilaterals. Suggest the idea of subsets if they know some set vocabulary.

ACTIVITY 2: Write in the word "triangle" or "quadrilateral" on the proper diagram on a given worksheet.

Suggested materials:

Worksheet X, pencil, paper, straight edge

Directions to student:

a. Write the name "triangle" in each figure with three sides.
b. Write the name "quadrilateral" in each figure with four sides.
c. Do you have more than one name in any of the figures?
d. On your paper list all the triangles. Use capital letters for their names.
e. On your paper list all the quadrilaterals. Use capital letters for their names.
f. From any one corner in figure C, connect all the other corners in the figure with line segments. (Use straight edge.)
How many triangles are in the figure?
g. From any one corner in figure J, connect all the other corners in the figure with line segments. (Use straight edge.)
How many triangles are in the figure?
h. Do the same thing in figures W, K, N, and R that you did in (f) and (g).

Suggested strategies:

a. Have the students note that both pairs of triangles F and G, as well as S and T, form a quadrilateral.
b. It may be desirable to use the word "vertex" instead of "corner" in items (f) and (g).
c. Attempt to have students note the relation between number of sides and number of triangles in any polygon.

ACTIVITY 3: Choose the triangles and quadrilaterals from a set of polygons projected on a screen.

Suggested materials:

One or more transparencies with models of polygons including a variety of triangles and quadrilaterals, as well as other polygons.
Overhead projector, screen, pencil, paper

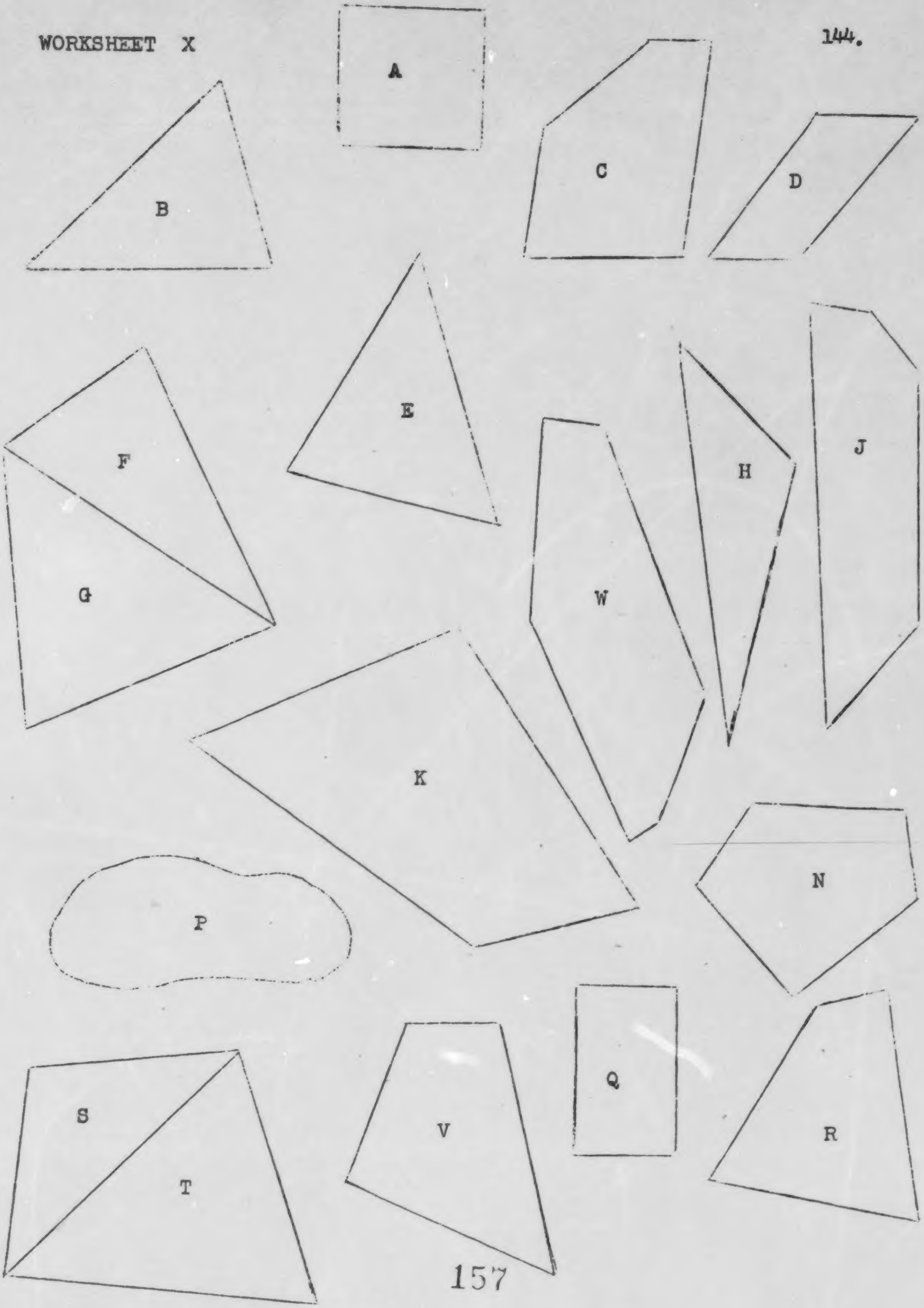
Directions to student:

a. List by letter all the quadrilaterals that you see on the screen.
b. List by letter all the triangles that you see on the screen.

Suggested strategies:

a. If the students have had previous experience in selecting triangles and quadrilaterals, it may be well to stress speed in choosing them this time.
b. To introduce a more complex aspect, choose models with overlapping triangles and quadrilaterals.

WORKSHEET X


A
B
C
D
E
F
G
H
J
W
K
N
P
Q
R
S
T
V

COMPETENCY: Classify simple plane figures by distinguishing some of their properties

OBJECTIVE:

The student will be able to find the measure of the third angle of a triangle when the measure of two of the angles are known.

ACTIVITY 1:

Suggested materials:

protractor, ruler, paper, pencil

Directions for student:

Draw any triangle on your paper. Make it large enough so that you can easily measure two of its angles with your protractor. Subtract the sum of those two angles from 180°. This will be the measure of the third angle. Check by measuring with your protractor.

Suggested strategies:

Have the students make different sizes and shapes of triangles.

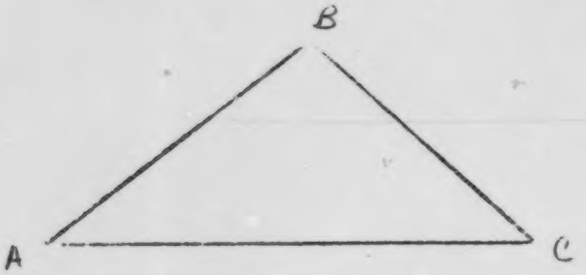
ACTIVITY 2:

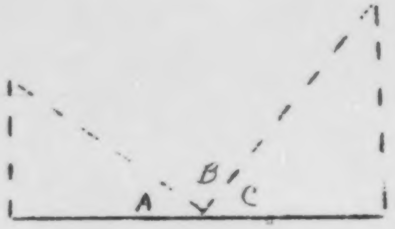
Suggested materials:

scissors, paper, pencil

Directions for the student:

Cut out any size triangle. Fold one angle over to the opposite side. Then fold each of the other angles in to this one looking like this:



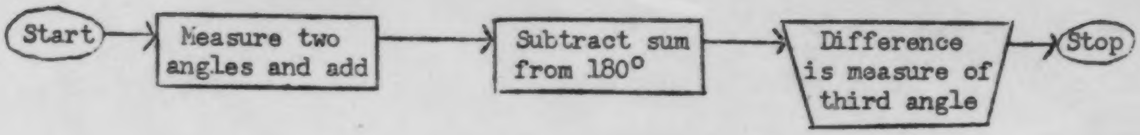


$m\angle A + m\angle B + m\angle C = 180^\circ$

ACTIVITY 3:

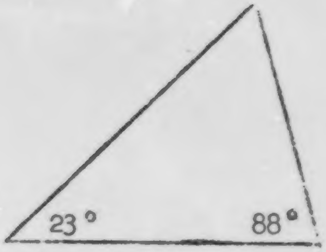
Suggested materials:
paper, pencil

Directions to student:
The flow chart is:

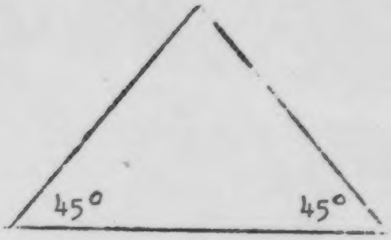

Start
Measure two angles and add
Subtract sum from 180°
Difference is measure of third angle
Stop

Using the flow chart, find the missing angle measures for these triangles.

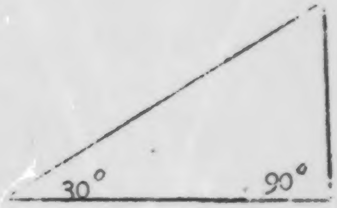
1.



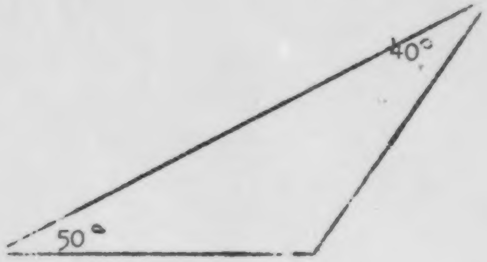
2.



3.



4.



5. $m\angle A = 65^\circ$ and $m\angle B = 42^\circ$
$m\angle C =$ ________

6. $m\angle S = 22^\circ$ and $m\angle T = 52^\circ$
$m\angle V =$ ________

COMPETENCY: Compute the perimeter of a given polygon

OBJECTIVE: By use of measuring instruments the student should exhibit the ability to determine the perimeter of plane geometric polygons.

ACTIVITY: Measuring to find the perimeter of plane geometric polygons

Suggested materials:

1. foot ruler
2. yard stick
3. tape measure
4. geoboard
5. graph paper
6. several models of plane geometric figures such as triangles, parallelograms, trapezoids, pentagons, etc.

Directions to student:

1. Measure and record the lengths of the edges of the following:
    1. The surface of the door
    2. The floor of the classroom
    3. The chalkboard
    4. Athletic fields
    5. Any plane polygon your teacher may indicate
2. Find the sum of the lengths of each object you have measured.
3. This sum represents the total dista nce around the object measured and is called the PERIMETER.

Suggested strategies: (for teacher)

1. As an added benefit, where possible, have the student develop formulas for finding perimeters of various simple plane geometric figures.
2. Indicate to students that this may be an individual or small group project in or outside the classroom.
3. The geoboard may be used to great advantage in demonstrating the perimeter of plane polygons.

COMPETENCY: Geometry--Compute the perimeter of given polygons.

OBJECTIVE:

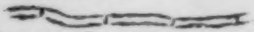
The student demonstrates his ability to compute the perimeter of polygons through estimation and the use of standard units and by formula.

ACTIVITY 1: Using a string to find a perimeter

Suggested materials: pen, pencil, unruled paper, pieces of string (marked in "even" units), plane figures cut from plastic cloth (naugaheid) or cardboard--two rectangles, two squares, two triangles of different sizes or shapes.

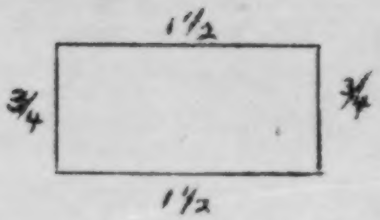
Directions to student:

(Part A)

1. Study the word per̲imeter with your instructor and/or classmates.
2. Make a list on the board, or on a transparency, of other words containing--metor, e.g. speedometer. What common thing do all these gadgets do?____________________
Then what is a perimeter?____________________________
3. Put one of your shoes, or your foot, on a piece of unruled paper and trace its outline in pencil, and then go over it with ink.
4. Study the outline and think of a way to find how long it is all around. Make a list on the board, or on a transparency, of the ways suggested by the class and discuss these suggestions.
5. If you decide to use a piece of string, notice that it has been marked off into equal units. 
How many units does it take to go around the outline?
____________________ units.
6. When you measure the length of the string on a ruler, how long is the outline of your shoe?
____________________ inches.
7. What is the mathematical name for this length as it relates to the outline?____________________________
8. Conclusion: The perimeter of your shoe is__________inches. Keep this paper to use in the next unit.

(Part B)

1. Make a drawing by tracing each of five different plane figures on a piece of unruled paper. Place them apart from each other and neatly so.
2. Using a string, measure the perimeter of each of these figures.
3. Under each figure write the word perimeter and list its length in inches as is done in this example. Mark the length of each side on the figure. How do you find the total length for the perimeter?______________________



Perimeter is 4 ½ inches

Suggested strategies:

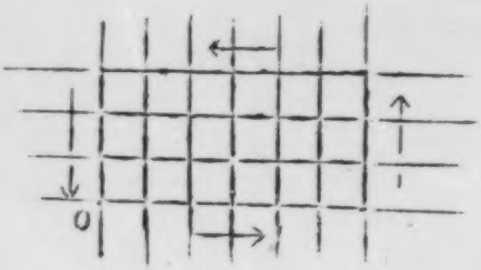
1. Keep these papers to compare them with the results obtained in Activity 2 and Activity 3.
2. If the geometric plane figures are cut from naugahsid they will not bend or break, and when soiled, can be put in a mesh bag and run through with a regular washing.

ACTIVITY 2: Using coordinate or graph paper to find perimeter

Suggested materials:

Duplicated copies of the two rectangles, and the two squares used in Activity 1, which are now on coordinate or graph paper. (¼ inch squares preferred.)

Directions to student:



1. Can you find the perimeter of this figure without using a string, or a ruler? Explain. ______________________
____
2. What is the perimeter of this rectangle in the units used on this paper? ________________ units.
3. If each unit measures ¼ inch, what is the perimeter in inches? ______________________ inches.

4. Above each figure write its name. Below the figure write "perimeter" and state the length in units and in inches.
5. Is the procedure you used to find the perimeter accurate? Why or why not? Explain.

6. In the rectangle used as an example, identify the width ____________units, the length ____________ units.
7. How many times did you use the measure of the width? ____________ the measure of length______________ in the total perimeter.
8. Then _________ x width plus _________ x length equals the perimeter or 2 widths + 2 lengths = perimeter.
9. If we substitute W for the width, L for the length, and p for the perimeter we have 2 w + 2 l = p   or p = 2 w + 2 l
10. 2(w + l) is called the formula for finding the perimeter of a rectangle.
11. Now study the 2 square figures. What do you notice about the width and the length?______________________ ____
12. So if we call the side of the square "s," what would represent the sum of all the sides? s + s + s + s = ___
13. What special name do we have for the sum of the lengths of the sides of a figure?___________________________
14. What can we use for the formula for the perimeter of a square? ____p = ____________________________
15. In the case of the triangle, measure the length of each side with a ruler. If you call the sides a, b, and c then p = a + b + c. This is the formula for a triangle.
16. Find (or compute) the length of the perimeters in the following problems after stating the formula for each problem.
    a. A square that measures 2 inches in every direction.
       Formula:_____________ perimeter:_____________

    b. A triangle whose sides are 3, 4, and 5 inches.
       Formula:_____________ perimeter:_____________inches

    c. A rectangle that is 5 inches wide and 7 inches long.
       Formula:_____________ perimeter:_____________inches

COMPETENCY: Compute the area of a rectangle and of a triangle

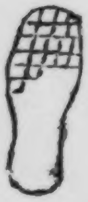
OBJECTIVE: Given a rectangle and a triangle, the student will be able to compute the area of each figure.

ACTIVITY 1: Using small geometric figures as square units to determine area.

Suggested Material: Unruled paper, pen, pencil and small plastic squares, rectangles, and triangles

Directions to Student:

(Part A)



1. Use the outline of your shoe which you drew in the Activity on Perimeter. If you do not have this, draw another outline in ink on unruled paper.
2. Cover the part inside the outline of the shoe with geometric figures one size and shape without overlapping them, but still put them close enough to touch each other. (In case there are not enough figures for all the students, take only 1 to 3 of the figures). As you lay them in place, trace carefully around each piece using pencil. See figure at left.
3. How many whole figures do you have inside the outline? ______
4. Call the space occupied by each figure one square unit. The number of square units is ______.
5. Think of all the little spaces not covered by a figure; if you could not get them together, how many more units do you think could be covered? ______
6. Now add this number to your answer in number 4. What is the total number of square units? ______
7. When you were computing the space occupied by the figure, you were using a square unit. This square unit is a unit of area. So when we speak of area we have in mind the space occupied by the figure.
8. What is the area of your foot in these square units?

   ______

(Part B)

1. Draw the outlines of two rectangles, two squares, and two triangles, different from each other either in size or in shape, on unruled paper. Do not put them close to each other.
2. Above each figure write its name--''rectangle'' or whatever it is.
3. Cover these outlines, one at a time with square unit figures and write under the outline the number of square units that are in it.
4. Do you think the number you have for the area is the exact value or only an approximate value? (Be careful in the case of the triangles) ______________

Suggested strategies:

1. Drawing the outline of all the figures in ink will make them permanent, and if the square units are traced in pencil the student is able to erase and re-arrange the unit-figures if he needs to.

2. The instructor may have to explain the meaning of approximate, and this activity offers a very good illustration of it.

3. If the instructor allows the students to choose from different size figures, the concept of "approximate" will be emphasized in Part a.

ACTIVITY 2: Using coordinate or graph paper to determine area

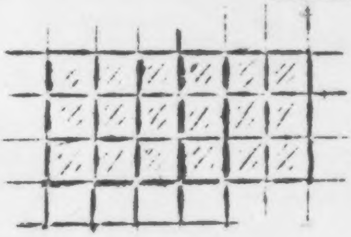
Suggested material:

Duplicated copies of the two rectangles, two squares, and the two triangles on coordinate or graph paper or individual grids. ($\frac{1}{4}$ inch is preferred), some scissors.

Directions to the student:

1. Can you find the area of this rectangle without using a square unit figure or a ruler?

Rectangle



Area is 18 sq. units

2. What is the area of this rectangle in terms of the square units used on this paper?
   ____________________ square units.

3. If each unit is $\frac{1}{4}$ inch in length, how many square units are there in one square inch.

4. Each square unit on the paper is 1/16 square inch in area.

5. How many square units are there in the rectangle?
   ____________________ square units.

6. What is the area of the rectangle in square inches?
   ____________________ square inches.

7. Above each figure on your paper write its name, as "Rectangle" in the example.

8. Under the figure write "Area is ______________" and fill in the information.

Suggested strategies:

If the student has difficulty in computing the area of the triangles:

1. Have him cut out a rectangle of graph paper.
2. Draw an inscribed triangle and then cut out the triangle.
3. Place the piece, or pieces, of the rectangle left over, on top of the triangle.
4. This should lead to the conclusion that the area of the triangle is $\frac{1}{2}$ the area of the rectangle.

COMPETENCY: Geometry--bisecting a line segment and an angle

OBJECTIVE: Given an angle, the student will be able to divide the angle into two equal parts.

ACTIVITY 1: Measuring with ruler

Suggested Materials: ruler, pencil, paper

Directions to Students:

1. With ruler and pencil, draw any angle on your paper with sides longer than 2 inches. Call it angle A. (figure 1)

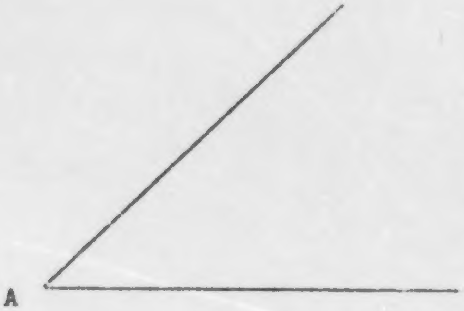


figure 1

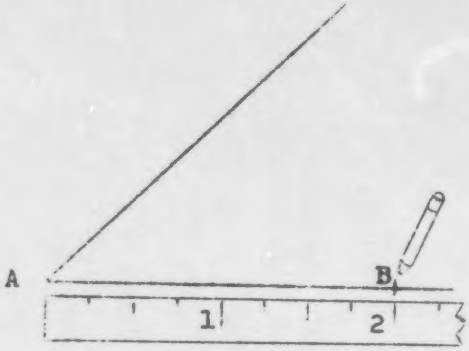


figure 2

2. Place the end of the ruler at A and mark a point 2 inches from A on one side of the angle. Call it point B. (figure 2)
3. Using the same process mark a point 2 inches from A on the remaining side. Call it point C. (figure 3) Note: AC and AB are the same length.

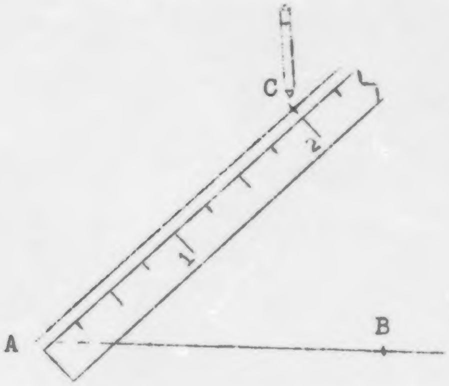


figure 3

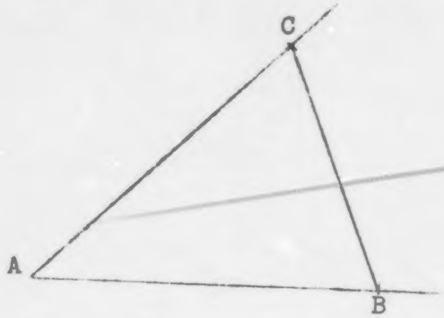


figure 4

4. Draw segment BC. (figure 4)
5. With your ruler find the length of segment BC.
6. Find 1/2 of the number found in step 5 and locate the point on segment BC. This is the midpoint of segment BC. Call it point D. (figure 5)

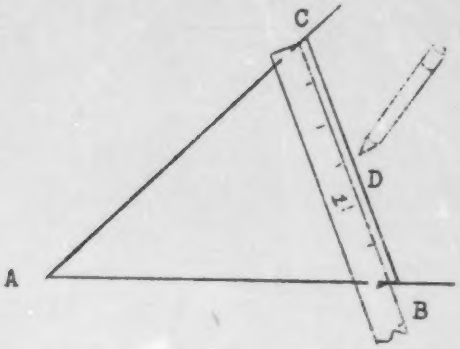


figure 5

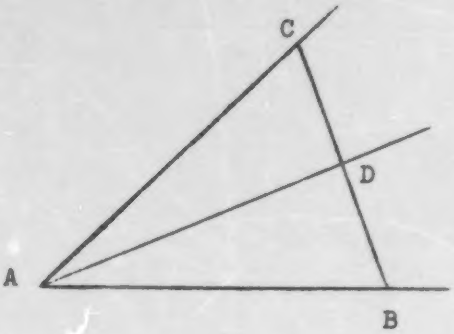


figure 6

7. With your ruler, connect points A and D. This is the bisector of the angle and divides the angle into two equal parts. (figure 6)
8. Using the same procedure, draw a different size angle and find its bisector.

Suggested Strategies:

1. Student must be able to find 1/2 of a given number.
2. Ruler should be graduated to 1/16 inch.
3. Encourage students to draw several angles and find the bisector of each.

ACTIVITY 2: Paperfolding

Suggested Materials: felt tip marker, ruler, wax paper

Directions to Students:

1. Take a piece of wax paper and with marker and ruler draw an angle of any size.
2. Label it as in the figure below.

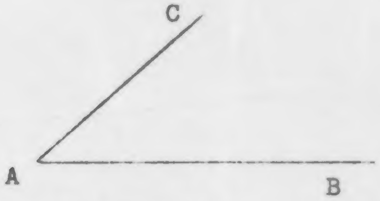


FIGURE 1

3. Fold the paper so that side AB falls exactly on side AC and crease the paper.
4. This line is the angle bisector and divides the angle into 2 parts.

Suggested Strategies:

1. Wax paper works nicely because creases show clearly, but other kinds of paper will work effectively.
2. Make sure that students match $\overrightarrow{AB}$ with $\overrightarrow{AC}$ and not point B with point C.

ACTIVITY 3: Use of Protractor

Suggested Material: paper, pencil, protractor

Directions to Students:

1. With straight side of protractor draw an angle of any size. Name it angle O. (figure 1)

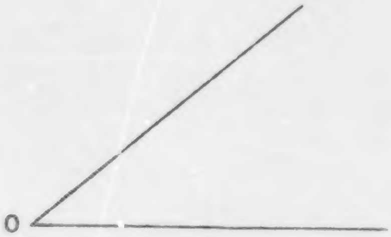


figure 1

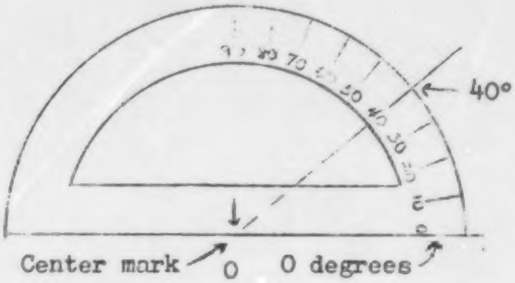


figure 2

2. With protractor, measure the angle to the nearest degree. (figure 2) Note: locate the center mark of the protractor and place it on the vertex. Place the zero mark of one of the scales on one side of the angle and read the measure of the angle from the remaining side.
3. Find 1/2 of this number and mark it with pencil. Name it point P. (figure 3)

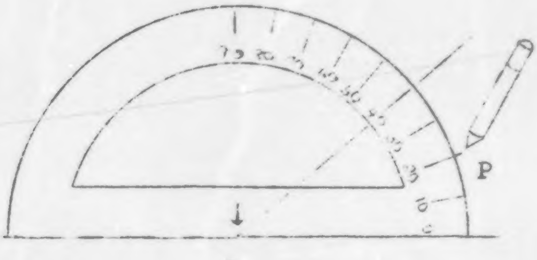


figure 3

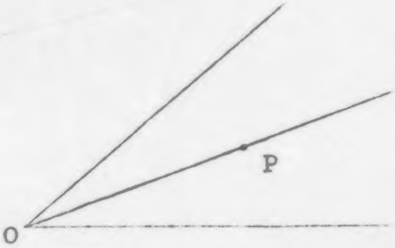


figure 4

4. Draw line OP. This is the angle bisector and it divides the angle into 2 equal parts. (figure 4)
5. Draw several angles of different sizes and locate the angle bisectors.

Suggested Strategies:

1. The student should have some knowledge of how to use the protractor.
2. Encourage students to try several examples on their own using different sized angles.

ACTIVITY 4: Use of compass and ruler

Suggested Material: pencil, ruler, compass

Directions to Students:

1. With ruler and pencil, draw an angle of any size on your paper. Name it angle W. (figure 1)

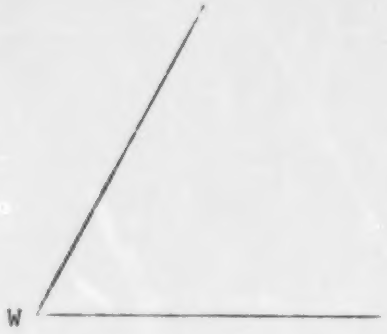


figure 1

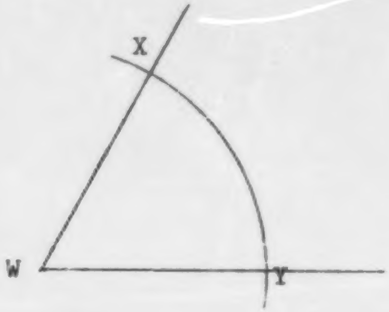


figure 2

2. Open your compass to any convenient radius and place the tip (metal point) at point W. Make an arc that crosses both sides of angle W. Name those points X and Y. (figure 2)
3. You may now change your compass radius or leave it as is. Place the tip at point X and make an arc inside the angle. (figure 3)

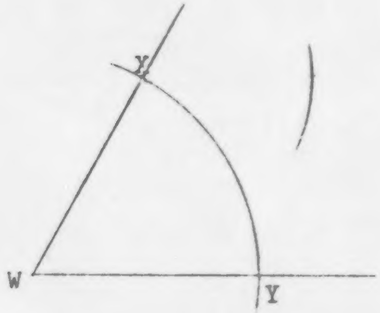


figure 3

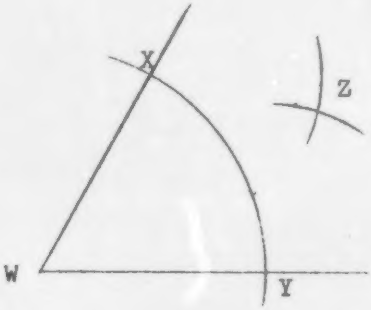


figure 4

4. Using the same compass radius as in step 3, place the tip at point Y and make an arc that crosses the other arc. Name it point Z. (figure 4)
5. Draw line WZ. This is the angle bisector and divides the angle into 2 equal parts. (figure 5)

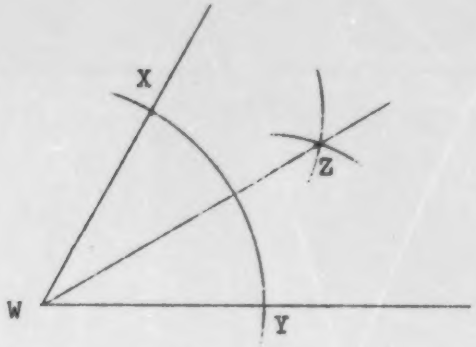


figure 5

6. Draw several angles of different sizes and locate the angle bisectors.

Suggested Strategies:

1. Student should have some knowledge of how to use a compass.
2. Use a compass that doesn't slip once the radius has been fixed.
3. Encourage students to try several examples.

COMPETENCY: Geometry--bisecting a line segment and an angle

OBJECTIVE: Given a line segment, the student will be able to locate the midpoint.

ACTIVITY 1: Measuring to find the midpoint

Suggested Materials: pencil, ruler, mimeographed sheet (worksheet accompanying activity)

Directions to Students: Use worksheet A and follow instructions.

Suggested Strategies:

1. Rulers should be graduated to 1/16 inch.
2. Students should be able to work with fractions.
3. Encourage students to draw segments of their own and locate the midpoint of each.
4. Answers to worksheet: (2) 3, 1 1/2 (3) 6 1/2, 3 1/4 (4) 3 3/4, 1 7/8 (5) 4 1/8, 2 1/16 (6) 2 1/4, 1 1/8 (7) 2 5/8, 1 5/16.

ACTIVITY 2: Paperfolding to find the midpoint

Suggested Materials: felt tip marker, ruler, wax paper

Directions to Students:

1. Take a piece of wax paper and with ruler and marker draw a line segment of any length on it.
2. Label one end point A and the other end point B.
3. Fold the paper so that endpoint A falls exactly on endpoint B and crease paper.
4. The point where the crease crosses segment AB is the midpoint.

Suggested Strategies:

1. Wax paper works nicely because it is transparent, and upon creasing will show lines very clearly.
2. Other kinds of paper will also work effectively.
3. Instead of using a felt tip marker to draw a segment on the wax paper, the student may make a crease in the paper to obtain the initial segment.

ACTIVITY 3: Use of compass and ruler to locate midpoint

Suggested Materials: paper, pencil, ruler, compass

Directions to Students:

1. Take a sheet of unlined paper and with ruler and pencil draw a line segment of anywhere from 1 to 4 inches long. (figure 1)

figure 1

figure 2

2. Label one endpoint P and the other endpoint Q. (fig. 2)
3. Place the tip of the compass (metal point) on P.
4. Open the compass over half way to point Q.
5. With the lead (pencil point) make a large arc on both sides of the segment. (figure 3)

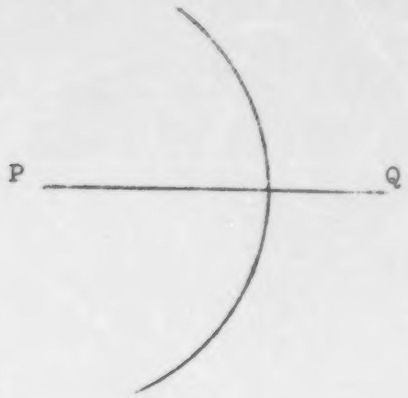


figure 3

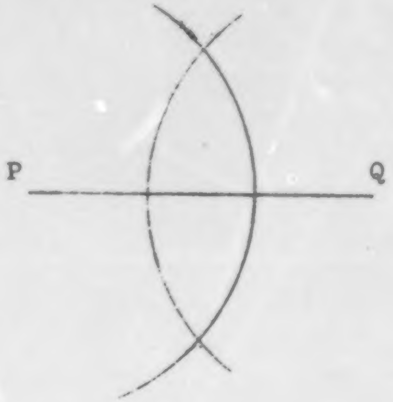


figure 4

6. Without changing the compass, place the tip on Q and draw an arc that crosses the first arc both above and below the segment. (figure 4)
7. Label these points X and Y. (figure 5)

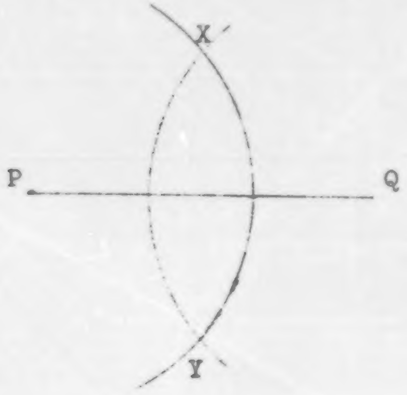


figure 5

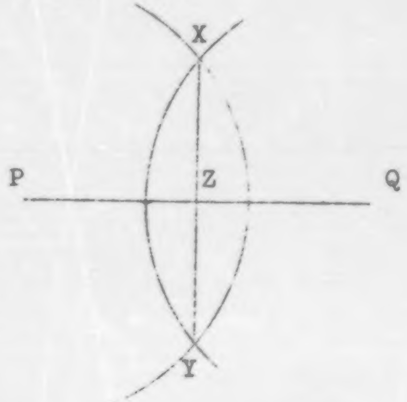


figure 6

8. With ruler and pencil draw segment XY. (figure 6)
9. Point Z where segment XY crosses segment PQ is the midpoint of segment PQ.
10. Note that segment XY is also called a perpendicular bisector of segment PQ.
11. Try this procedure on other line segments.
12. Explore these possibilities:
    a. What happens if you don't open the compass wide enough? (At least half the length of the segment)
    b. What happens if the arcs don't cross in two places?
    c. What happens if your segment is over 15 inches long?
    d. What happens if your segment is very small (less than 1/4 inch)?

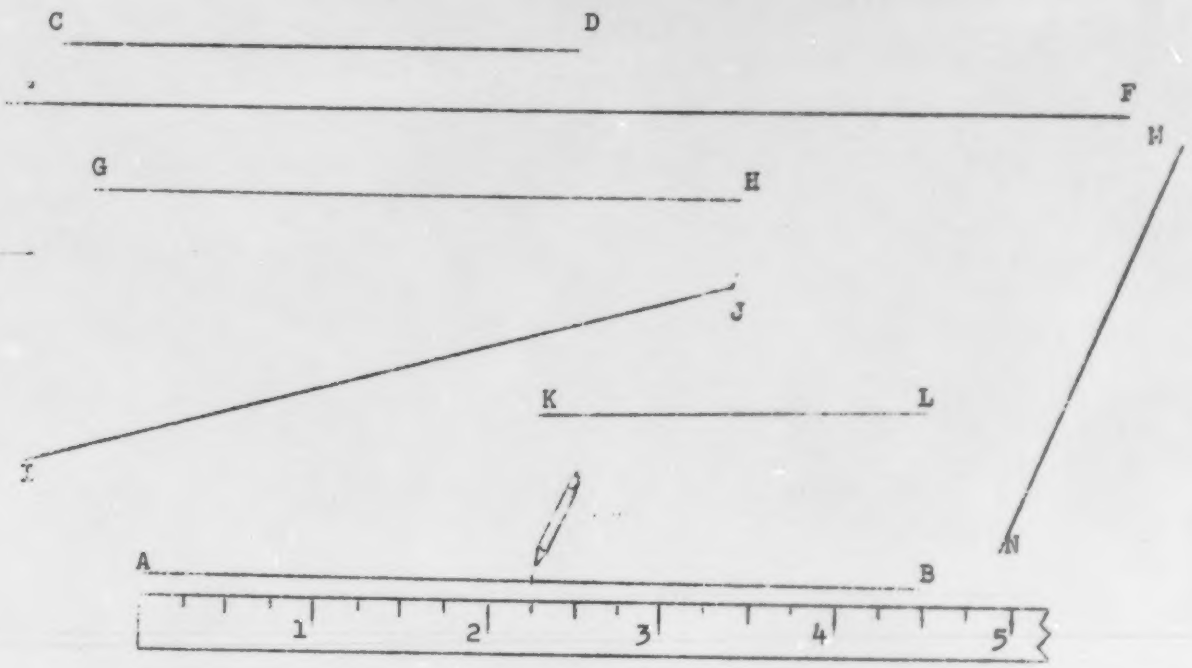
Suggested Strategies:

1. Be ready to assist students if the following problems occur:
    a. Arcs do not intersect
    b. Student has changed the compass radius
    c. The segment is too short or too long to be practical
    d. The arcs intersect in one point but not two, either at the midpoint or only above the segment or only below the segment
2. Encourage students to try several examples.
3. Use a compass that doesn't slip once the radius has been fixed.
4. This is the construction for a perpendicular bisector and any line through the midpoint of $\overline{PQ}$ would serve as a bisector of $\overline{PQ}$.
5. Students should have some knowledge of how to use a compass.

Worksheet A

On this sheet you see 7 line segments.

1. With your ruler, measure the length of each segment and record it in the 2nd column of the chart.
2. Next, calculate 1/2 of each of those numbers and record each in the 3rd column.
3. With your ruler, use the 3rd column number and locate a point on each segment equal to that measure.
4. For example: segment AB has length 4 1/2. 1/2 of 4 1/2'' is 2 1/4''. Place the end of the ruler at point A and mark a point on segment AB 2 1/4'' from A. This is the midpoint of AB.



	segment	length of segment	1/2 of length of segment
1	AB	4 1/2	2 1/4
2	CD		
3	EF		
4	GH		
5	IJ		
6	KL		
7	MN		

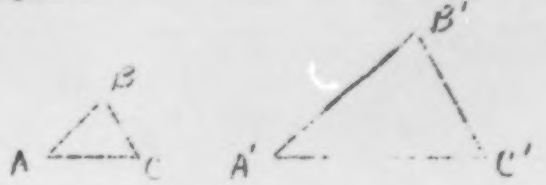
COMPETENCY: Identify the conditions of similarity of triangles and use the properties of similarity to solve problems.

OBJECTIVE

The student will be able to recognize similar triangles and solve problems relating to similar triangles.

ACTIVITY 1: Triangles which have the same shape care called similar triangles.

Example 1:

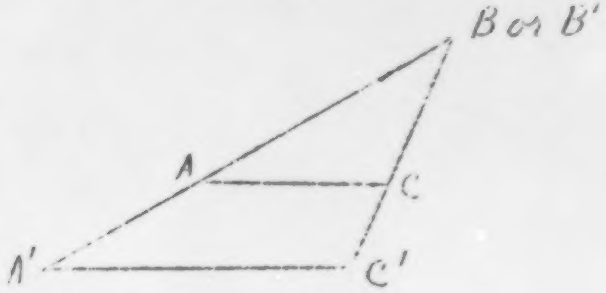


$m\ A = m\ A',\ m\ B = m\ B',\ m\ C = m\ C'$

and $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$

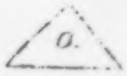
meaning the corresponding angles have the same measure and the corresponding sides have the same ratio.

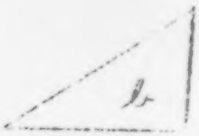
Example 2:

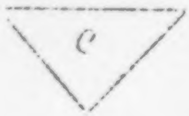


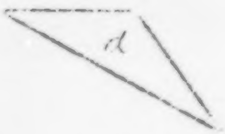
$ABC \simeq A'B'C'$

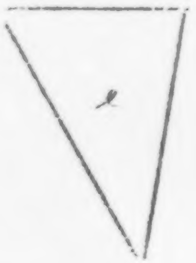
Which of the following are similar triangles? ______________





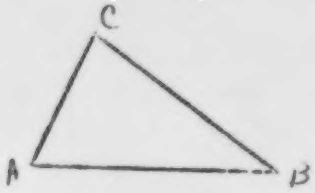


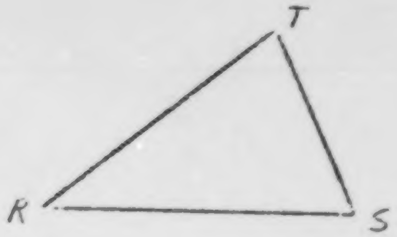




ACTIVITY II: Suggested materials: unlined paper, scissors, tracing paper

Directions for Student: These triangles are similar. Find the corresponding angles. Read entire activity before starting to answer questions.

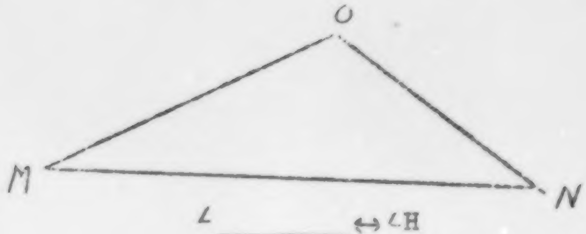


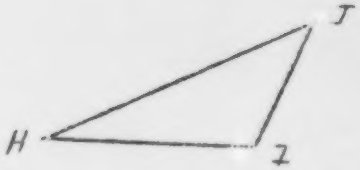


∠A ⟷ ∠______

∠B ⟷ ∠______

∠C ⟷ ∠______





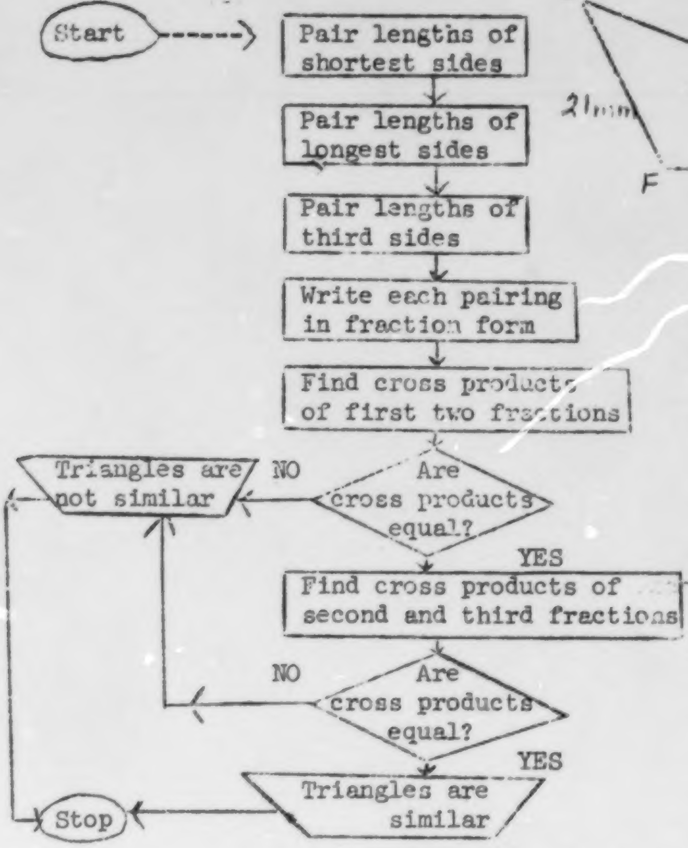
∠______ ↔ ∠H

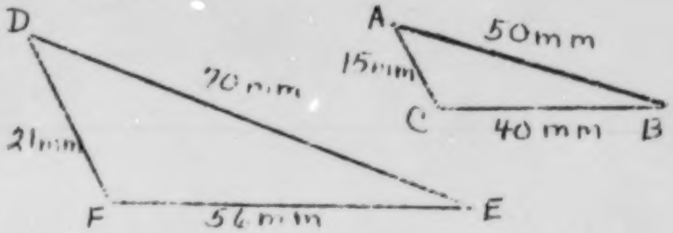
∠______ ↔ ∠I

∠______ ↔ ∠J

Trace one triangle of each pair. Cut it and mark the letter on each angle. Now fit the angles on the other one of the pair and fill in the name of the angle.

ACTIVITY III: If the lengths of the sides of two triangles are known, the flow chart below dexcribes a way of determining whether or not the triangles are similar.


Start
Pair lengths of shortest sides
Pair lengths of longest sides
Pair lengths of third sides
Write each pairing in fraction form
Find cross products of first two fractions
Are cross products equal?
NO
YES
Triangles are not similar
Find cross products of second and third fractions
Are cross products equal?
NO
YES
Triangles are similar
Stop


D
70 mm
21mm
F
56 mm
E
A
50mm
15mm
C
40 mm
B

15 ↔ 21, 50 ↔ 70, 40 ↔ 56.

$\frac{15}{21}$ $\frac{50}{70}$ $\frac{40}{56}$

$\frac{15}{21} \times \frac{50}{70}$ 21 x 50 = 1050
15 x 70 = 1050

YES

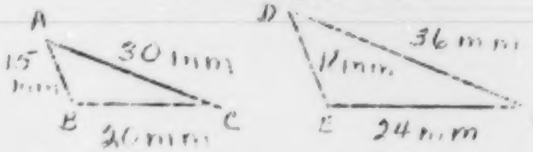
$\frac{50}{70} \times \frac{40}{56}$ 70 x 40 = 2800
50 x 56 = 2800

YES

△ABC ~ △DEF

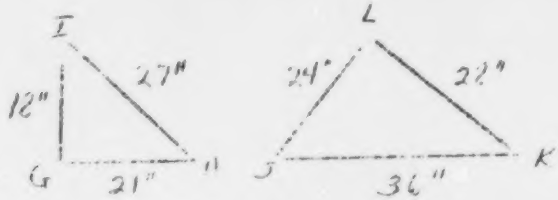
Use your flow chart to determine which of the following pairs of triangles are similar. Circle the correct answer.

1.


A
15 mm
30 mm
B
20 mm
C
D
36 mm
17mm
E
24 mm
F

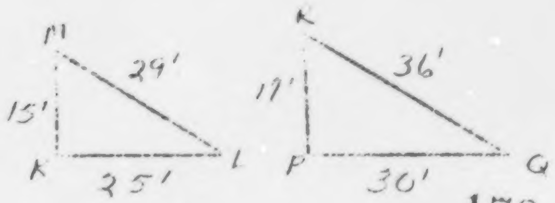
△ABC is similar to △DEF
△ABC is not similar to △DEF

2.


I
12"
27"
G
21"
H
L
24"
28"
J
36"
K

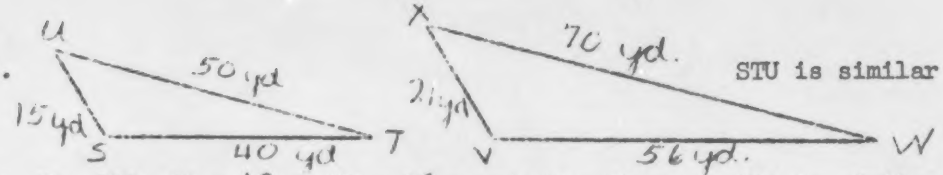
△GHI is similar to △JKL
△GHI is not similar to △JKL

3.


M
29'
15'
K
25'
L
R
36'
17'
P
30'
Q

△KLM is similar to △PQR
△KLM is not similar to △PQR

4.


U
50 yd
15 yd
S
40 yd
T
X
70 yd.
21 yd
V
56 yd.
W

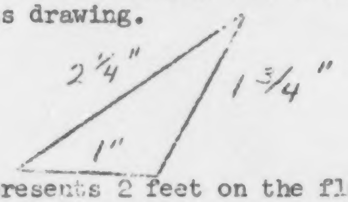
STU is similar to VWX

5. In MNO, NO = 48 m, MN = 36 m and OM = 52 m
In RST, RS = 45 m, ST = 60 m and TR = 65 m

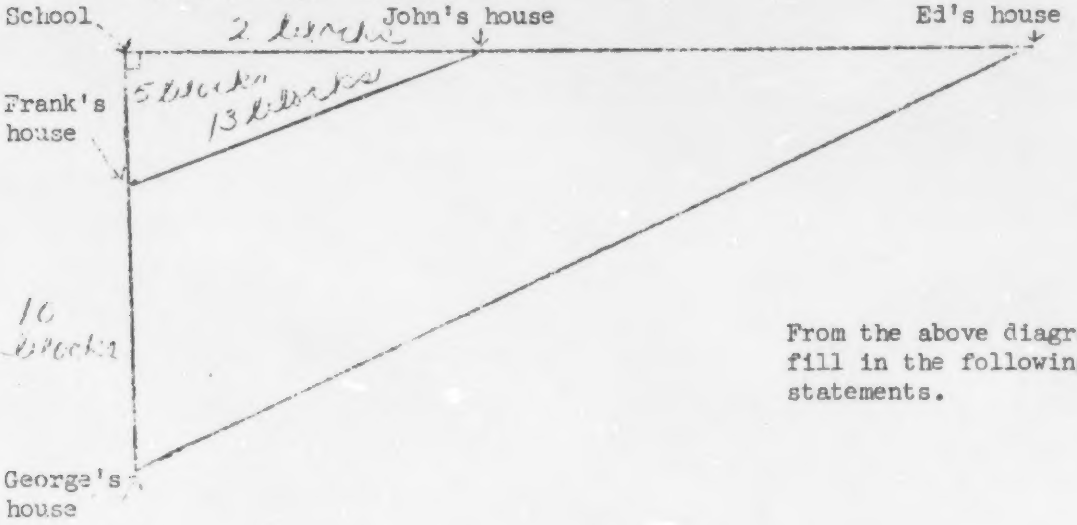
MNO is similar to RST
MNO is not similar to RST

ACTIVITY IV:

1. Your father has drawn a sketch of a flower bed around which he plans to put a fence. This is his drawing.



2. One inch on this sketch represents 2 feet on the flower bed. Find the length of each side of the flower bed and how many feet of fence he will need.


School
2 blocks
John's house
Ed's house
Frank's house
5 blocks
13 blocks
10 blocks
George's house

From the above diagram fill in the following statements.

George's house to school = ________ blocks
John's house to Ed's house = ________ blocks
Ed's house to school = ________ blocks
Ed's house to George's house = ________ blocks

3. Two planes leave Chicago at the same time. One is flying northeast at 475 m.p.h. and the other is flying southeast at 625 m.p.h. At the end of one hour they are approximately 785 miles apart. At the end of two hours about how far will they be apart? (Suggestion: Make a sketch first)

COMPETENCY: Using mathematics as a tool in performance of his duties. Geometry--Classify simple plane (and solid) figures by distinguishing some of their properties.

OBJECTIVES:

1. The student will be able to select a cube from a set of geometric solids.

2. The student will be able to identify the six congruent faces, the twelve straight-line edges and the eight vertices of a cube.

ACTIVITY 1: Recognizing cubes

Suggested materials:
Set of geometric solids, including cubes, square prisms, parallelopipeds and others. Examples of cubes, e.g., children's blocks, dice, commercial cubical blocks, SOMA blocks, etc. Projectuals for overhead.

Directions to student:

1. A square has four equal sides and four right angles. Find a solid that has some square faces.

2. Find a solid that has six faces, all of which are squares. This is called a cube.

3. Are there any other cubes in the collection of solids?

4. Two faces meet at an "edge." Locate the edges of your cube. How many are there?

5. Three edges meet at a "vertex." Locate the vertices of your cube. How many are there?

Suggested strategies:

1. Use commercial cubical blocks or SOMA blocks to further develop concepts and recognition of cubes.

2. Use projectuals to show what a two-dimensional view of a cube would be.

OBJECTIVES: The student will be able to determine the two-dimensional patterns that can be used to construct a cube, and will construct one or more cubes from these patterns.

ACTIVITY 1: Constructing cubes.

Suggested materials:
Tagboard, scissors, ruler, compass, transparent tape, projectuals for figures 2 and 3.

Directions to student:

1. Construct or draw six congruent squares from tagboard. Tape together to form a cube.

2. Cut enough edges to allow pattern to lie flat. How many cuts must be made? What restrictions are there on the cuts?

3. Draw patterns of figures 2 and 3. Cut out, fold along marked edges, tape together to attempt to form cubes.

Suggested strategies:
Try to determine the conditions on the patterns that will result in cubes. Are there some rules that govern the patterns?

Evaluation: Use figure 4 for a quick quiz.

Challenge: There are eleven patterns that will give cubes. Can you find all of them? (see Figure 5)

(Note: A similar activity might be developed around solids with rectangular faces, or with square and rectangular faces. Other activities of a similar nature might suggest themselves.)

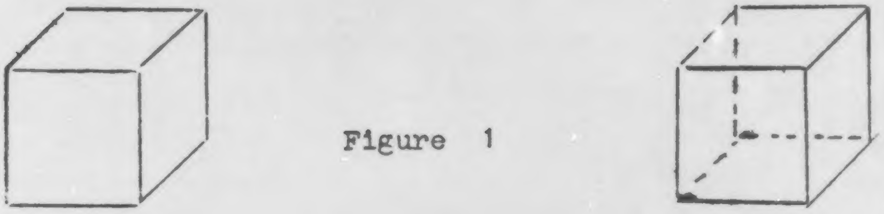


Figure 1

a

b

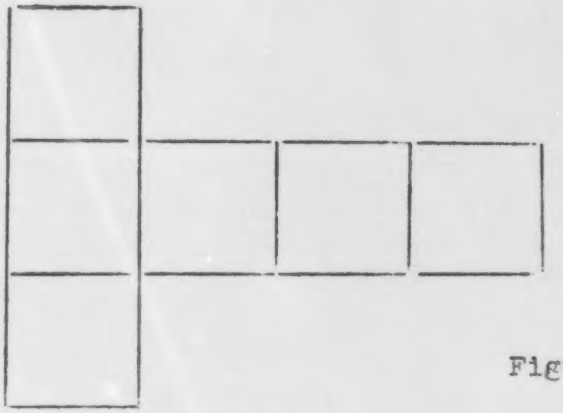


Figure 2

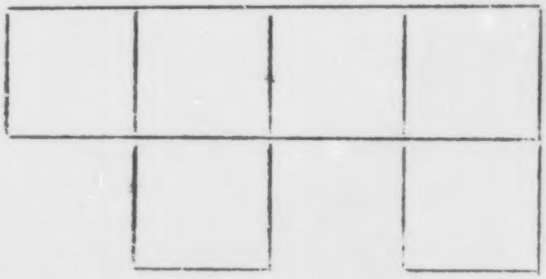
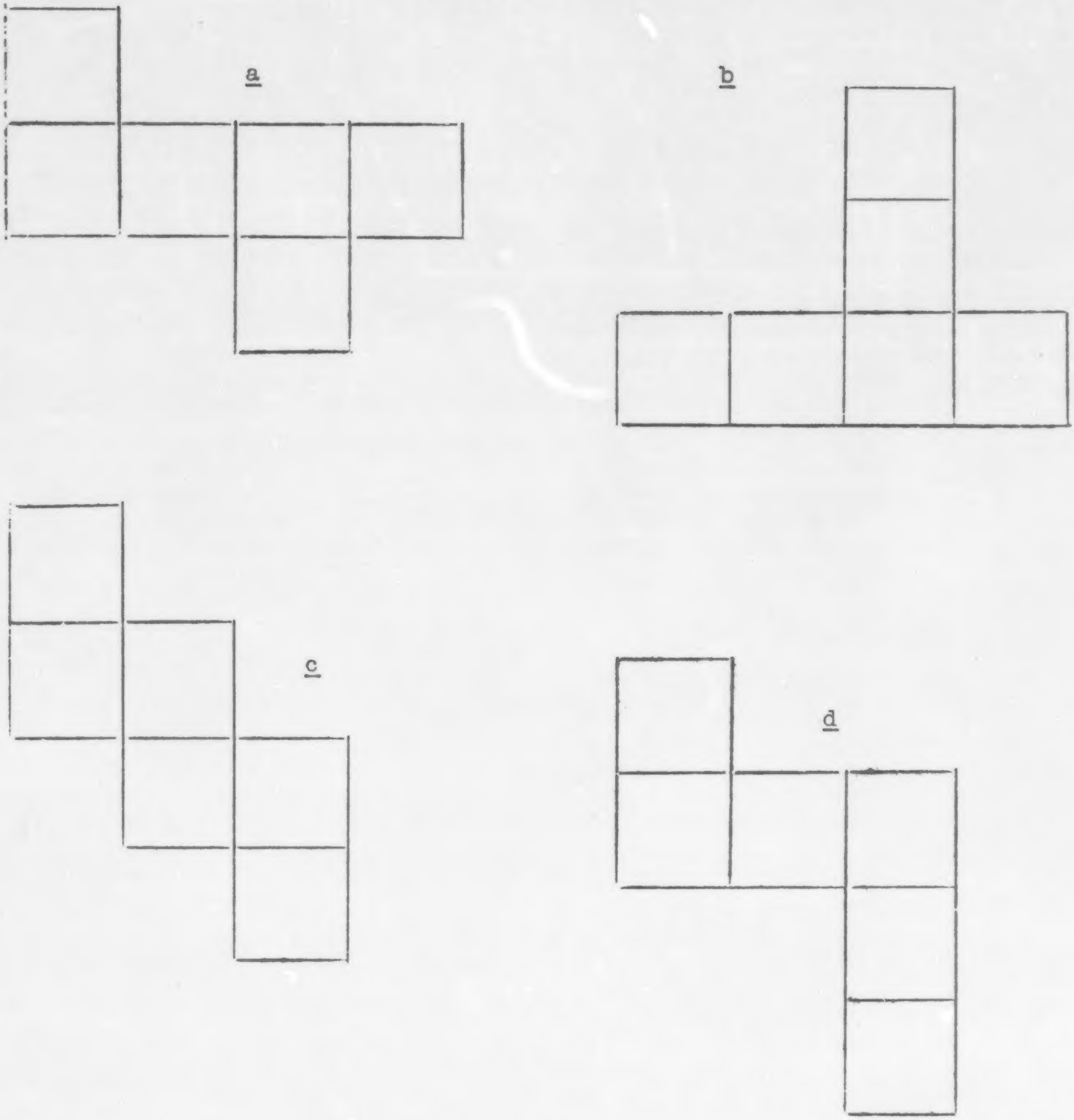
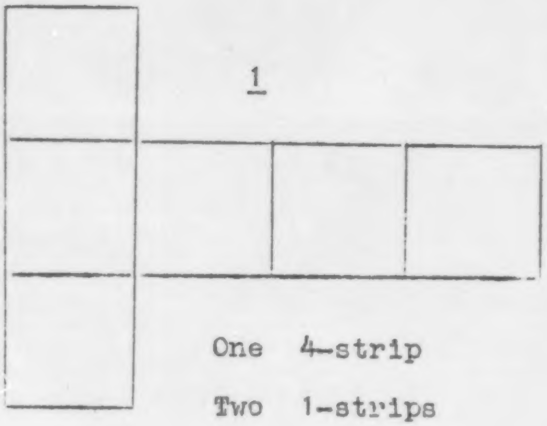


Figure 3

Figure 4

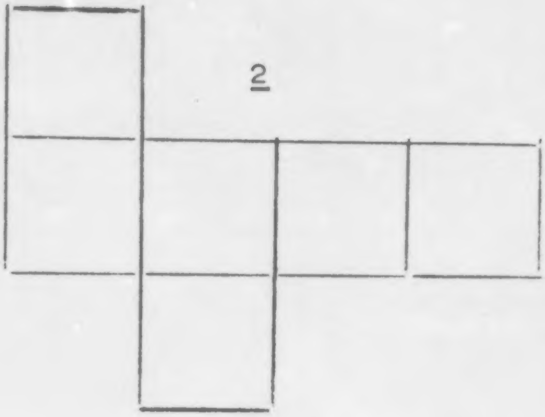




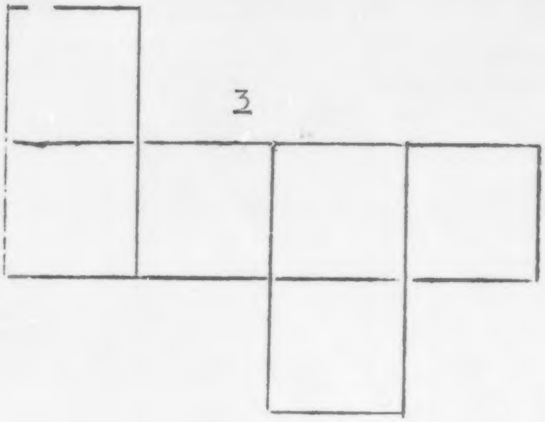
1

One 4-strip

Two 1-strips



2



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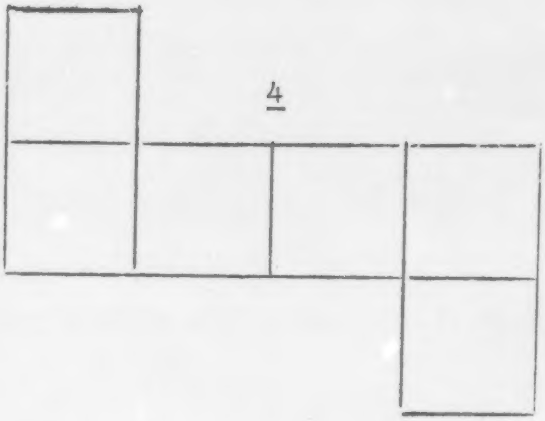

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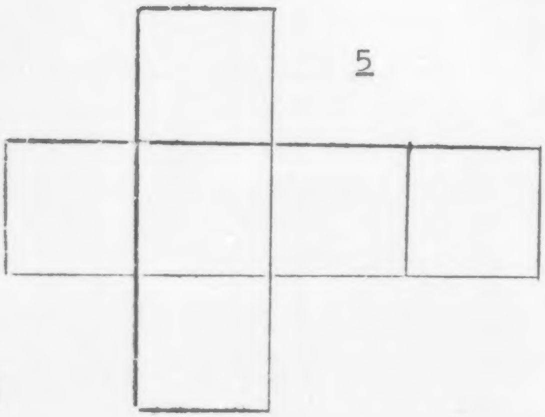

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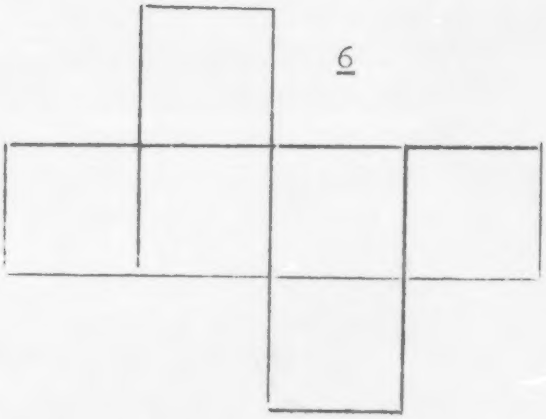


6

COMPETENCY: (In addition to the first part of the unit) -- Measurement
Use standard measuring devices of length, area and volume to make measurements.

OBJECTIVES:

The student will be able to find the length of the edges, the areas of the faces, and the volumes of selected cubes.

ACTIVITY 1: Measuring cubes

Suggested materials:

Set of cubes of various sizes, ruler, materials for computation

Directions to student:

1. Use the ruler to find the length of edges of cube.
   What will be the total length of all the edges?
   (See the first section of the unit.)

2. What is the area of one of the square faces?
   ($A = e^2$)
   What is the total surface area of the cube?

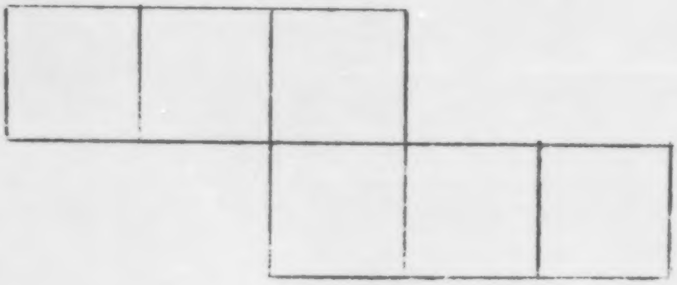
3. What is the volume of the cube?
   ($V = e^3$)

Suggested strategies:

Compare areas and volumes when a second cube has an edge length twice that of a first cube; three times that of a first cube, etc.

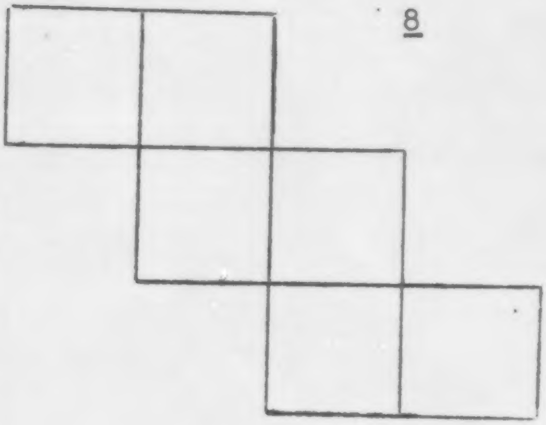
Two 3-strips

7



Three 2-strips

8

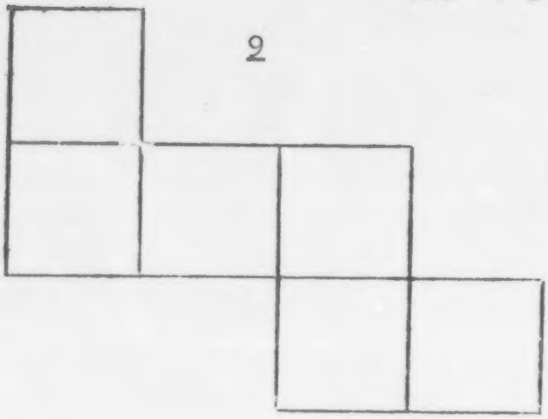


One 3-strip

One 2-strip

One 1-strip

9



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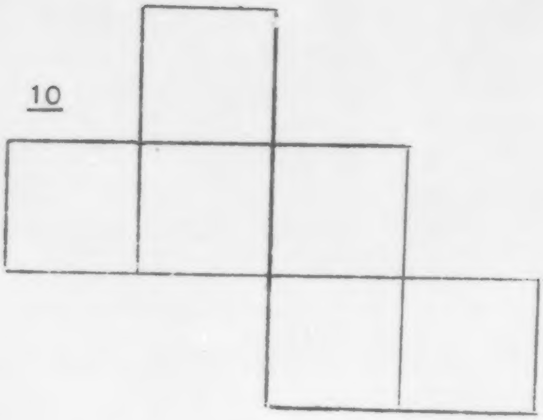

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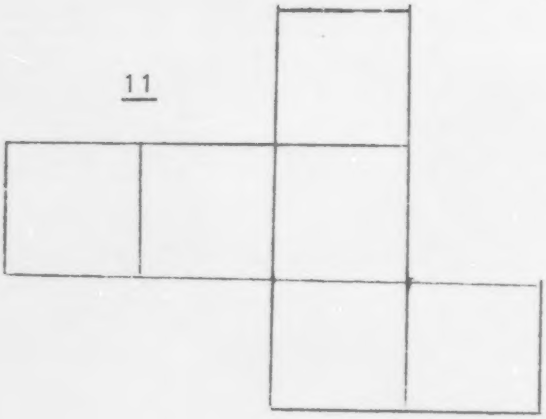


COMPETENCY: Apply the common English measures of length, area, volume, weight, and temperature.

OBJECTIVE: Given an object to be measured. The student can give its length--accurate to the nearest unit of measure.

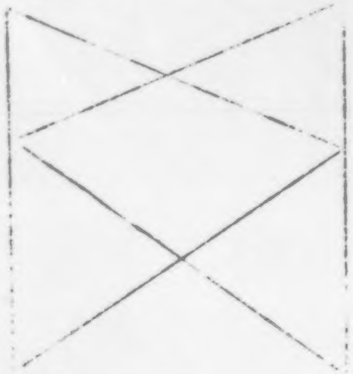
ACTIVITY 1:

Give each student some drawings and objects to be measured using the "rule of thumb". That is, using the last joint of the thumb as the unit of measure.

Suggested materials:

Suggested objects for measurements are books, desks, the teacher's desk, or papers with drawings such as

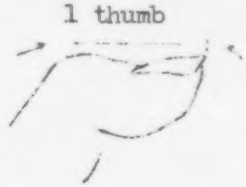
or



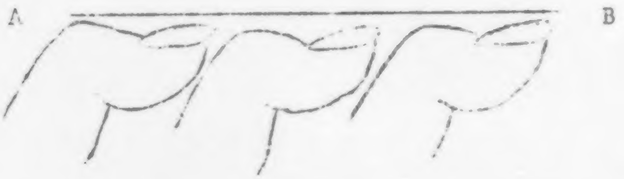
Drawn in any scale the teacher desires.

Directions to student:

Today we are going to answer the question, "How long is this line?" To tell "how long," you can count with your thumb. You can use the end of your thumb like this.



To count "how long" line A B is, place your thumb like this:



A B is 3 thumbs long.

"How long" are the lines on the paper?
"How long" are the sides of the objects on your desk?
When you have finished, check your answers with your friends.

Suggested strategies:

Teachers, use this last "check" and the variance it shows to bring up the idea of an inch as the same unit for us. The above activity can be varied with the use of dried beans, paper clips, or other objects as the standard of measure. The teacher need only vary the illustrations accordingly.

COMPETENCY: Apply the common English measures of length, area, volume (dry or liquid), weight, time, money and temperature.

OBJECTIVE: Given an object to be weighed the student can give an answer accurate to the nearest unit of measure.

ACTIVITY 2:

Give each student a balance and a box of paper clips to use as his standard measure. Ask him to find the weight in p.c. (paper clips) of the objects provided.

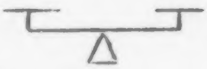
Suggested materials:

Box of paper clips, balance, objects to be measured--ink eraser, chalk, five pieces of paper, 25 index cards, wooden cubes, etc.

Directions to student:

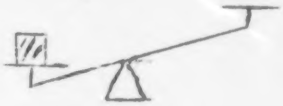
Today we are going to answer the question, "how heavy is this?" To begin our work we are going to use the paper clips to answer "how heavy." All we need do is put the object we are checking on one side of our balance and put enough paper clips on the other side to get both trays to be the same height.

1.



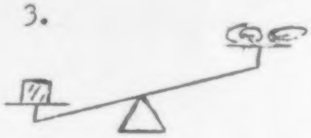
empty

2.



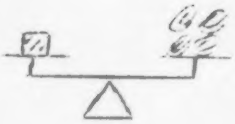
How heavy is our box?

3.



It is heavier than 2 paper clips.

4.



Our box is 4 paper clips heavy.

Now see "how heavy" all the things are that your teacher gives you.

Suggested strategies:

If possible have different students use different size paper clips and measure the same objects. Their results will easily lead to a need for a uniform unit of measure which you can introduce as the ounce. The above activity can be varied in many ways by using different units of measure--such as lima beans, navy beans, poker chips, etc.

COMPETENCY: Apply the common English measures of length, volume (dry or liquid), weight, time, money and temperature.

OBJECTIVE: Given a surface to be measured, the student can give its area to the nearest unit of measure.

ACTIVITY 1: Each student will measure the area of the drawings given him by using trading stamps as his standard unit.

Suggested materials:

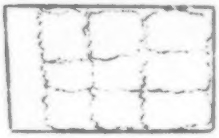
Envelope enclosing at least 100 trading stamps (To avoid problems with the stamps sticking on everything they could be stuck to paper then cut to shape.)

Ditto sheets on which are drawn rectangles and other shapes whose area can be measured.

Directions to student:

Today we will see how many pieces will fill the inside of a drawing. Since we will need pieces that are the same size, we will count with trading stamps. In the envelope you have been given, you will find the trading stamps you will need. Use them to fill in the drawings on your sheet of paper. Write down each answer as soon as you find it.

1.



1. will hold 9 stamps, but since there is some of the inside which is not covered, we can say that our answer is 9 or 10 trading stamps.

Suggested strategies:

If possible try to have your students use different brands of stamps. The variation in size will lead to a discussion of a need for a standard unit and the introduction of square inch.

The above activity can be also presented with the use of dry lima beans as your standard measure or canceled postal stamps.

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ACTIVITY 2: The student will measure object in two units and attempt to detect the patterns.

Suggested materials:

yardsticks, rulers, ditto with list of objects to be measured (A suggested list is included in the student's directions.)

Directions to student:

Today we will be using the yard stick and ruler. Since we now know how to measure with these guides we will try to find a way to change an answer in feet into an answer in inches and so on. Use the yard stick and ruler to fill in the following table. Try to find how the feet answers change into inches, and yard answers change into feet. If you think you see how the numbers are changing ask your teacher to see if you have the right answers.

Measure	Yard	Foot	Inch
length of textbook	X	___ ft.	___ in.
width of textbook	X	___ ft.	___ in.
length of your desk	X	___ ft.	___ in.
width of your desk	X	___ ft.	___ in.
height of a window	___ yd.	___ ft.	X
width of a window	___ yd.	___ ft.	X
length of the room	___ yd.	___ ft.	X
width of the room	___ yd.	___ ft.	X

Suggested strategies:

The above list may be lengthened to best fit the needs of your pupils for a variety of objects. It is the author's opinion that no object should be measured in all three units because of the confusion created. Of course, this opinion is relative to the previous experiences and abilities of your students.

ACTIVITY 3:

Suggested materials:

Dittoed copies of the list which follows.

Directions to student:

Circle the longer measurement of each pair below.

1. 14 inches   2 feet
2. 4 feet   3 yards
3. 9 inches   8 feet
4. 16 inches   1 foot
5. 2 feet   12 yards
6. 3 inches   1 foot
7. 36 inches   4 feet
8. 12 feet   2 inches
9. 4 yards   9 feet
10. 12 feet   1 yard

Suggested strategies:

This activity is described as it would be used on an intuitive approach under which the pupil need not be able to demonstrate his method for selecting his choice, if the activity follows a previous discussion of changing units--the pupil would be expected to demonstrate his method of selection.

The ten problems are by no means all inclusive and should be lengthened if your student shows any hesitation on an intuitive choice.

COMPETENCY: Convert, using tables, English to Metric measures and conversely.

OBJECTIVE: Given a measurement of length in the English or Metric systems, the student will be able to convert to the other system.

ACTIVITY 1: The student will use his ability in measuring in the different systems to fill in a conversion table of the standard units.

Suggested materials:
ruler, yard stick, meter stick

Directions to student:
Having studied how to measure by inches, feet, yards and by millimeters, centimeters, and meters, we will use this skill to fill in the following table. Be sure to go slow and make your measurements as carefully as you can.

English	Metric
1 inch	___ cm
1 foot	___ cm
1 yard	___ cm = ___ m
___ inch	1 cm
___ inch	1 meter
___ yard	1 meter

Now fill in these blanks with what you have just discovered:

3 inches	___ cm
2 yards	___ m
___ inch	3 cm
___ inch	2 m
___ yard	2 m
6 inches	___ cm
___ inch	10 cm

Suggested strategies:
As accuracy is the essential point of this activity it may be wise to have each student check his work with you as soon as the first six measurements are completed.

COMPETENCY: Convert, using tables, English to Metric measures and conversely.

OBJECTIVE: Given a measurement of weight in the English or Metric system the student will be able to covert to the other system.

ACTIVITY 1: The student will measure standard units of one system in the units of the other system. (English and Metric)

Suggested materials:

scale, weights (ounces, pounds), weights (grams, kilograms)

Directions to students:

Today we will discover how ounces, pounds, grams and kilograms are related. To do this we will use the scale to measure the different weights and then, since these measurements are a little tricky, we will compare our answers.

To begin, fill in this table.

English	Metric
1 oz.	___ gm (gram)
___ oz.	___ kilogram

Now ask five of your neighbors for their answers. Add these and divide by 5. This will give you an average answer. Check with the teacher to see how close you have come to the set answer, then finish this table.

English	Metric
3 oz.	___ gm
6 oz.	___ gm
___ oz.	4 kilogram
___ lb.	10 kilogram

Suggested strategies:

The above activity must be preceded with a discussion of the need for slow work and accuracy.

COMPETENCY: Recognize that no measurement is precise.

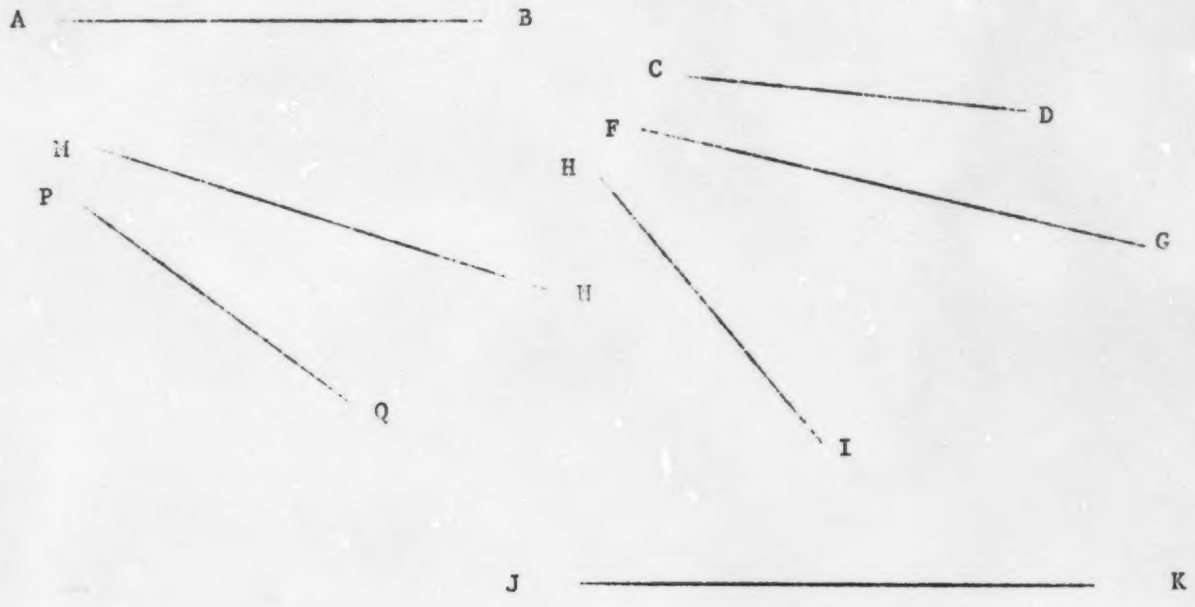
OBJECTIVE: Given a compass the student will determine which lines are longer than a given line.

ACTIVITY 1: Comparing lengths of lines.

Suggested materials:
Compass, handout sheets and pencils

Directions to student:
Bob wants to be sure that all the fish he has caught are above the legal minimum length. Using only a compass, compare the lengths of his fish (line $\overline{AB}$, $\overline{CD}$, etc.) against the legal minimum length ($\overline{L.E}$)

legal length = L ______________________________ E



COMPETENCY: Recognize that no measurement is precise.

OBJECTIVE: Given seven line segments, a group of students will each measure the items, accurate to the nearest 1/16 of an inch and nearest millimeter, record this information and then compare the results.

ACTIVITY 1:

Measuring and comparing the lengths of line segments

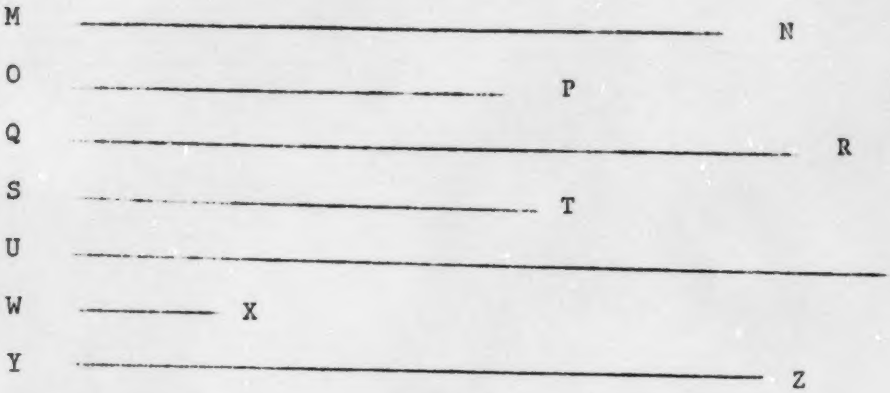
Suggested materials:

Handout sheet including the line segments and table; rulers (English and Metric)

Directions to student:

Today you are to work in three steps.

1. Measure the line segments on your paper using inches accurate to the nearest 1/16. Write your results in the table which is also on your paper.
2. Measure the same line segments using the metric ruler and writing your answer accurate to the nearest millimeter.
3. Compare your results with three other students.



Line Segment	Inches	Centimeter, Millimeter
MN		
OP		
QR		
ST		
UV		
WX		
YZ		

Suggested strategies:

After the students have finished their measurements and comparisons, develop their ideas on why their answers are not all identical into a discussion of the concept of being "precise."

COMPETENCY: Use metric units of length, mass and volume in making measurements.

OBJECTIVE: Given a set of objects, or drawings of same, the student will measure, calculate and report on volume expressed in metric units.

ACTIVITY: Use of measure to find volume.

Suggested materials:

Give each student some rectangular solids, drawings and a meter stick to be used in this activity.

Directions to student:

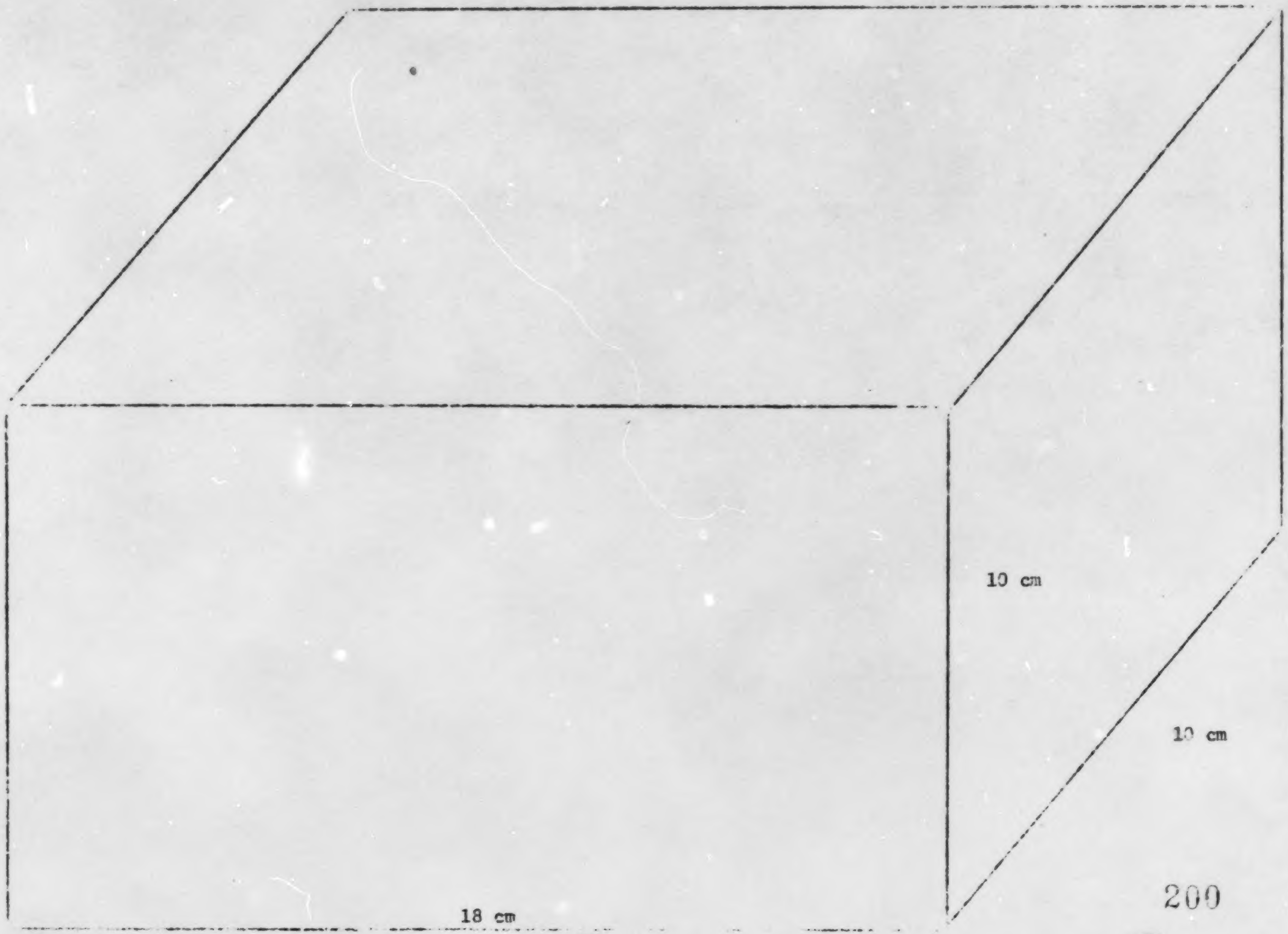
As you know, volume is calcuated by multiplying the length by the width and by multiplying that product by the height. For example the box below measures 18 cm x 10 cm x 10 cm.

18 cm length times 10 cm width = 180 cm^2
180 square centimeters
180 cm^2 x 10 cm height = 1800 cm^3 or
1800 cubic centimeters

On the next page you will find a box, measure the box yourself, to be sure you can use the meter stick.

Now measure the objects available and record the results below.

	Object	Length	Width	Height	Volume
1.					
2.					
3.					
4.					


10 cm
10 cm
18 cm

COMPETENCY: Use metric units of length, mass, and volume in making measure.

OBJECTIVE: Given a graduated millimiter measuring cup, the student will fill a liter container with water and compare his results with the following conversion tables.

milliliter (ml) = 1/1000 liter
centiliter (cl) = 1/100 liter
deciliter (dl) = 1/10 liter

ACTIVITY: Each student will have to fill a one liter container with water from a graduated milliliter cup and record his results.

Suggested materials:

recording sheet, water, a one liter container, and a graduated measuring cup

Directions to student:

Fill the graduated milliliter to the tenth line. You now have a centiliter of water. Pour this water into the container and repeat this process 9 more times. You now have 10 centiliters of water in the container. Now pour all the water in the graduated measuring cup. The 10 centiliters equal one deciliter. Mark this line on the measuring cup. Now how many of these deciliters do you need to fill the liter container?

Fill in the table below:

____ Deciliters = 1 liter
10 Centiliters = 1 Deciliter
____ Centiliters = 1 liter
10 Milliliters = 1 Centiliter
____ Milliliters = 1 liter

COMPETENCY: Use standard measuring devices of length, area, volume, time and temperature to make measurements.

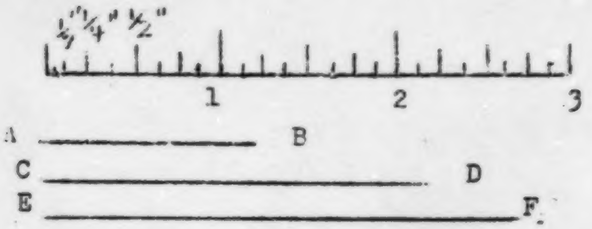
OBJECTIVE: Given a group of line segments the students will measure them accurate to the nearest inch, 1/2 inch, 1/4 inch, and 1/8 inch.

ACTIVITY: Measuring length in the English system.

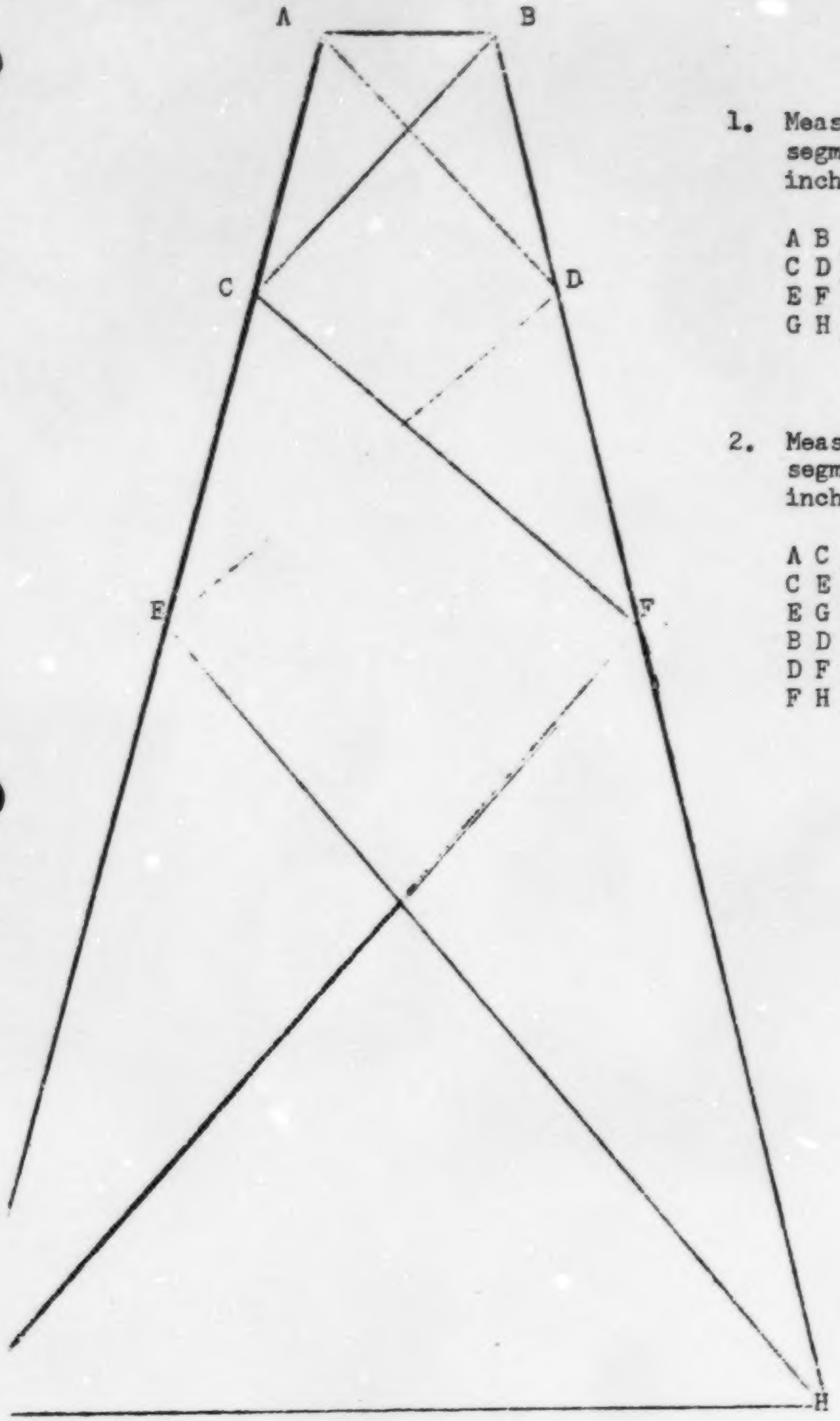
Suggested materials:
rulers and handout with the line segment drawings

Directions to student:
What is the length of line segment

A B ____________________
C D ____________________
E F ____________________



Have someone check your three measurements then go on to the drawings which follow:



1. Measure the following line segments to the nearest whole inch.

A B ____________________

C D ____________________

E F ____________________

G H ____________________

2. Measure the following line segments to the nearest half inch.

A C ___________________

C E ___________________

E G ___________________

B D ___________________

D F ___________________

F H ___________________

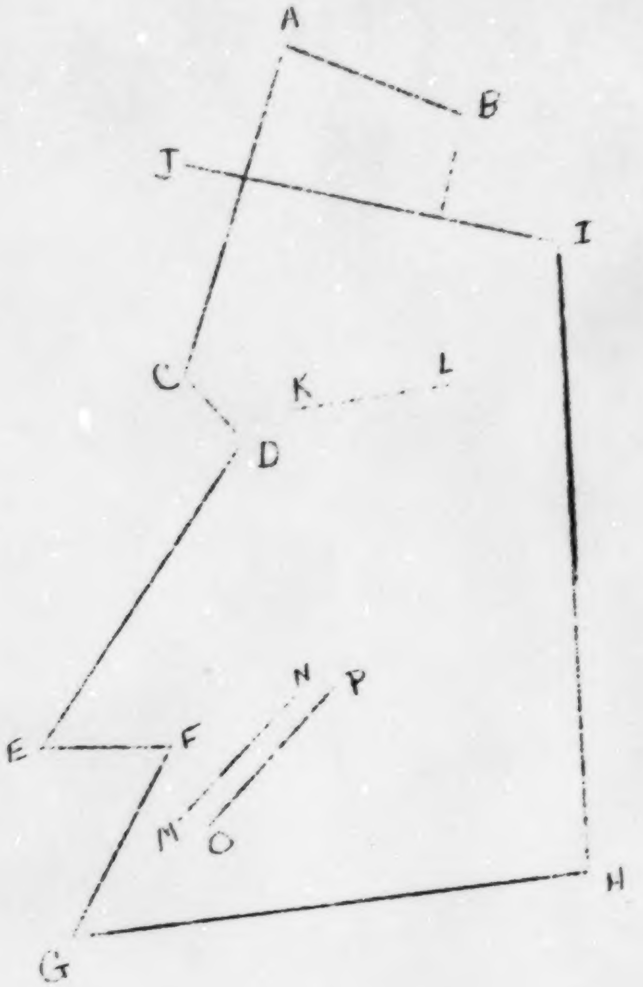
3. Measure the following line segments to the nearest quarter inch.

A D ____, C F ____, E H ____, C B ____, E D ____, G F ____

4. Measure all line segments to the nearest eighth of an inch.

A B ________
A C ________
C D ________
D E ________
E F ________
F G ________

G H ________
H I ________
I J ________
K L ________
M N ________
O P ________


A
B
J
I
C
K
L
D
N
P
E
F
M
O
H
G

COMPETENCY: Use standard measuring devices of length, area, volume, time, and temperature to make measurements.

OBJECTIVE: Given various scales the student will weigh objects, record their results, and answer the three questions stated below.

ACTIVITY: Weight by various scales

Suggested materials:
- bathroom scales (pound)
- scales for ounces (postal scales)
- scales for grams (science lab equipment)
- objects for weighing (books, erasers, etc.)

Directions to student:
On the recording sheet write down the weights of at least six objects and try to answer the questions at the bottom of the page.

Recording Sheet

Object	Bathroom Scale (lb)	Ounces	Grams
1.			
2.			
3.			
4.			
5.			
6.			

Question 1: Which scale is the least accurate one on which to weigh small objects?

Question 2: On which scale(s) can you read the results directly?

Question 3: Can you estimate, from the data in your table above, how many grams would be equal to 10 ounces?

COMPETENCY: Use standard measuring devices of length, area, volume, time and temperature to make measurements.

OBJECTIVE: Given thermometers of Fahrenheit and Celsius (Centigrade) scales, the student will take the temperature of six water samples and record the data in the given table.

ACTIVITY: Using standard themometers

Suggested materials:

thermometers (Fahrenheit and Celsius Scales)
containers for water, ice and water

Directions to student:

Today you should have some fun by using two different kinds of thermometers to measure the temperature of different glasses of water. Be sure to ask your teacher how long you should leave the thermometers in the water to get an accurate reading.
Fill in your results in the table below.

Water Samples	Fahrenheit	Celsius
ice water		
sample x		
room temperature water		
sample y		
boiling water		

Suggested strategies:

Teachers set your own limitations on how long the students should keep the thermometers in the water to be accurate without taking too much time.

COMPETENCY: Measurement

OBJECTIVES: The student will be able to measure and draw a line segment using a ruler.
The student will be able to draw and measure an angle of 90 using a protractor.
The student will be able to construct a rectangle and know properties of a rectangle.

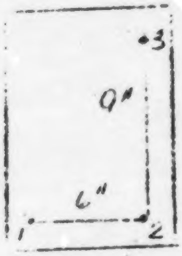
ACTIVITY 1:

Suggested materials:
Protractor, 12 inch ruler, pencil, paper

Directions to the student:

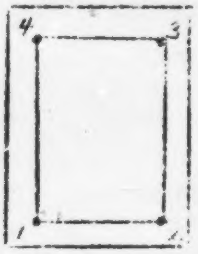
1. Use the ruler and draw a straight line 6 inches long at the bottom of your paper. Label the left end point 1 and the right end point 2.
2. Use the protractor and draw a line through end point 2 at an angle of 90°.
3. Mark a dot on this line at a distance of 9 inches from point number 2. (See diagram 1) Label this dot number 3.

Diagram 1:



4. Use the protractor and draw a line through point number 3 at an angle of 90°.
5. Use the protractor and draw a line through point number 1 at an angle of 90°. Label the point where this line meets the line in (4) as number 4. (See diagram 2.)

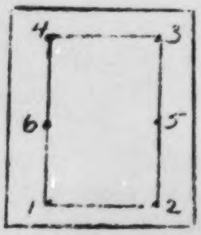
Diagram 2:



What is the measure of the angle formed by these two lines?

6. Use your ruler to find the point halfway between points 2 and 3. Label this point 5. Use your ruler to find the point halfway between points 1 and 4. Label this point 6. (See diagram 3.)

Diagram 3:



What is the distance between (a) points 2 and 5? ________
(b) points 4 and 6? ______________
(c) points 1 and 3? ______________
(d) points 2 and 4? ______________
(e) points 1 and 4? ______________
(f) points 3 and 4? ______________
Are any of these distances the same? ________________
If so, which ones? ______________________________

Suggested strategies:

The teacher may now ask the students to construct rectangles of other dimensions. Ask the same questions. The student should keep the rectangles he has constructed for use in the next unit.

ANSWERS TO QUESTIONS:

(a) 4 $\frac{1}{2}$ inches
(b) 4 $\frac{1}{2}$ inches
(c) 10 7/8 inches
(d) 10 7/8 inches
(e) 9 inches
(f) 6 inches

yes, (a,b) (c,d) (e,f)

COMPETENCY: Measurement

OBJECTIVES: The student will be able to draw and measure angles using a protractor.

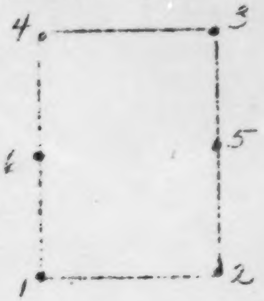
ACTIVITY 1: Drawing and measuring angles.

Suggested materials:
Protractor, 12 inch ruler, pencil, 6 x 9 inch rectangles drawn on paper

Directions to student:

1. Label the rectangles as in diagram 1. Five is the middle (midpoint) of the segment with endpoints 2 and 3. Six is the middle (midpoint) of the segment with endpoints 1 and 4.

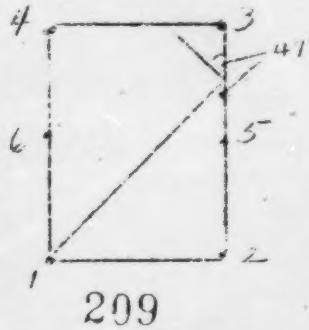
Diagram 1:



COMMENT--Diagram 1 is a model of a pool table. Pretend that the dots on the corners and the sides are the "pockets" of the pool table.
For our game, we will only use the cue ball and will try to determine the path that it will follow when hit.
You are to measure the angle with your protractor and draw the path the ball travels with your ruler. Continue drawing the path of the ball and measuring the angle the ball bounces off the side until the path leads to a pocket.

Rules:
1. The cue ball must always hit a side before it hits a pocket.
2. The ball will always bounce off a side at the same angle it hits. (See diagram 2)

Diagram 2:



2. Starting from corner pocket number 1, hit the ball at an angle of 45° with a side. Use the protractor to measure this angle and use the ruler to draw the path the ball travels.
3. Answer the following questions:
    1. How many sides will the ball hit before it hits a pocket?
    2. Will the same thing happen if you start from the same place but at a different angle? Explain your answer.
    3. Will the same thing happen if you start at a different position but at the same angle? Explain your answer.

ACTIVITY 2: Drawing and measuring angles

Suggested materials:
Same as Activity 1.

Directions to the student:

1. Move the ball one inch along either side from the corner pocket number 1.
2. Hit the ball at a 45° angle from the side and draw the path the ball travels.
3. Answer the following questions:
    1. How many sides will the ball hit before it hits a pocket?
    2. What happens if you move the ball one inch from the corner pocket number 1 but along the other side and then hit it at a 45° angle?

ACTIVITY 3: Drawing and measuring angles and distance between two points.

Suggested materials:
Same as Activity 1.

Directions to the student:

1. Place the ball at corner pocket number 1. Hit the ball so that it will bounce off one side and hit side pocket number 6.
2. Answer the following questions:
    1. What is the measure of the angle with which the ball hits the side?
    2. What is the distance from corner pocket number 2 to the point on the side hit by the ball?

ACTIVITY 4: Drawing and measuring angles and distance between two points.

Suggested materials:
Same as Activity 1.

Directions to the student:

1. Place the ball one inch from corner pocket number 1 along either side. Repeat activity 3 and answer the same questions.
2. Place the ball one inch from corner pocket number 1 but along the other side not used. Repeat Activity 3 and answer the same questions.

ACTIVITY 5: Measurement of angles. Inductive reasoning.

Suggested materials:
Same as Activity 1.

Directions to the student:

1. Place the ball at corner pocket number 1. Determine the path of the ball if you hit two sides and then hit side pocket number 6.
2. Answer the following questions:
    1. What are the measures of the angles with which the ball hits the sides?
    2. What is the distance from corner pocket number 3 to the point where the ball hits the first side?

Suggested strategies:
Vary the size of the "pool table." Use 6 x 10, 5 x 8, 4 x 7, etc. Allow the student to make up more complex "shots" but still answering the same type of questions.

ANSWERS:

Activity 1:
1. three sides

Activity 2:
1. It will never hit a pocket
2. The ball follows the same path but in the opposite direction.

Activity 3:
1. 70°   2. 2 $\frac{1}{4}$ inches

Activity 4:
1-1. 73° , 1-2. 2 3/4 inches
2-1. 68° , 2-2. 2 1/8 inches

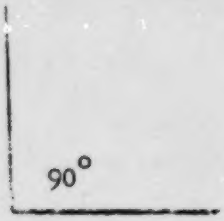
Activity 5:
1. 41° and 47°   2. 2 $\frac{1}{4}$ inches

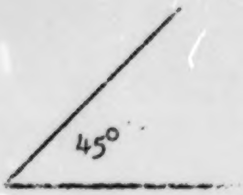
COMPETENCY: Round off measurements to the nearest given unit of the measuring device used such as rulers, protractors, and thermometers.

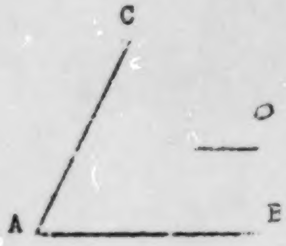
OBJECTIVE: Given various angles and geometric shapes the student will use a protractor to measure the angle to the nearest degree.

ACTIVITY 1: Each student is to use a protractor and measure all the angles on the following pages and answer two relevant questions.

Directions to student:
Using your protractor measure the four angles labeled below.

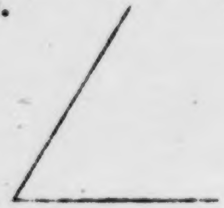




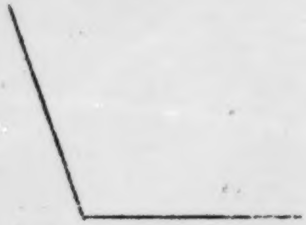


____°

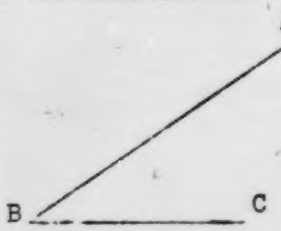
Write the number of degrees for angle C A B here.________________
Have your teacher check this before you proceed to measure the angles given below.



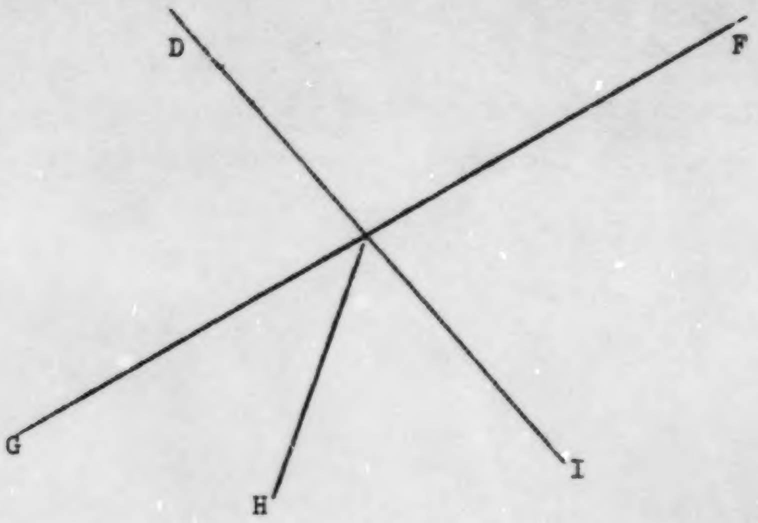
____________°



____________°



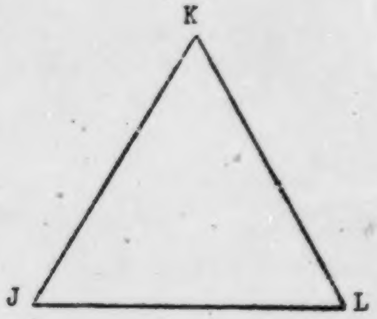
∠ A B C = ________°



201.

1. ∠ DEF = ______°
2. ∠ DEG = ______°
3. ∠ GEH = ______°
4. ∠ GEI = ______°
5. ∠ HEI = ______°
6. ∠ IEF = ______°

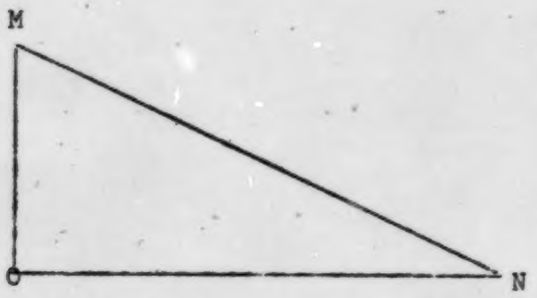
Are ∠ GEH + ∠ HEI = to ∠ GEI   yes / no ?

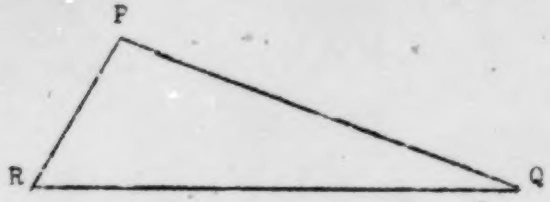


∠ JKL = ______________

∠ KLJ = ______________

∠ LJK = ______________





∠ MNO = ______________

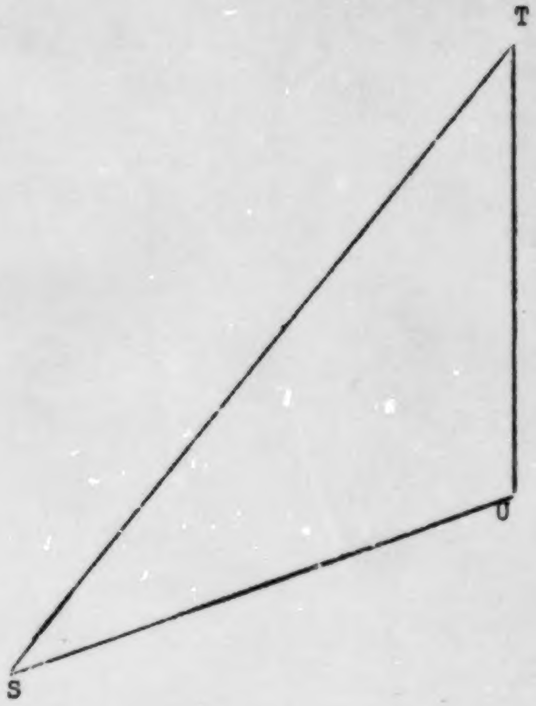
∠ NOM = ______________

∠ OMN = ______________

∠ PQR = ______________

∠ QPR = ______________

∠ PRQ = ______________



∠ STU = ____________

∠ TUS = ____________

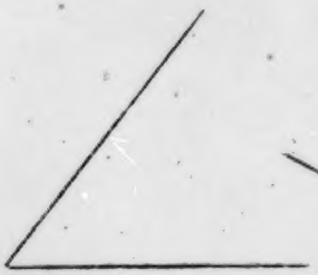
∠ UST = ____________

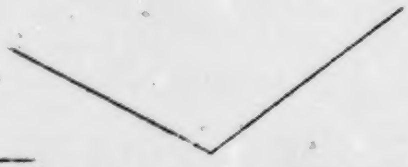
How many degrees do the three interior angles of a triangle equal.

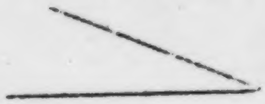
____

1. Use a protractor to find the measure of each angle.

A. __________ B. __________ C. __________







2. Use a protractor to construct angles with the indicated measures.

A. 38° B. 47° C. 121°

OBJECTIVE: Given a map with a list of cities and their respective population the students will round off this figure to the nearest hundred.

ACTIVITY: Rounding off given measures and answer two questions.

Suggested materials:

Give each student a work sheet, map and pencil.

Directions to student:

On the worksheet you will find 40 towns and cities listed. Round off the population of each town to the nearest hundred. You see that the population of Le Mars has been rounded for you. 1800

Please continue.

Which town is the largest in population? ____________

Which town is the smallest in population? ____________

OBJECTIVE: Given a printed centigrade and fahrenheit thermometer scales the student will fill it in.

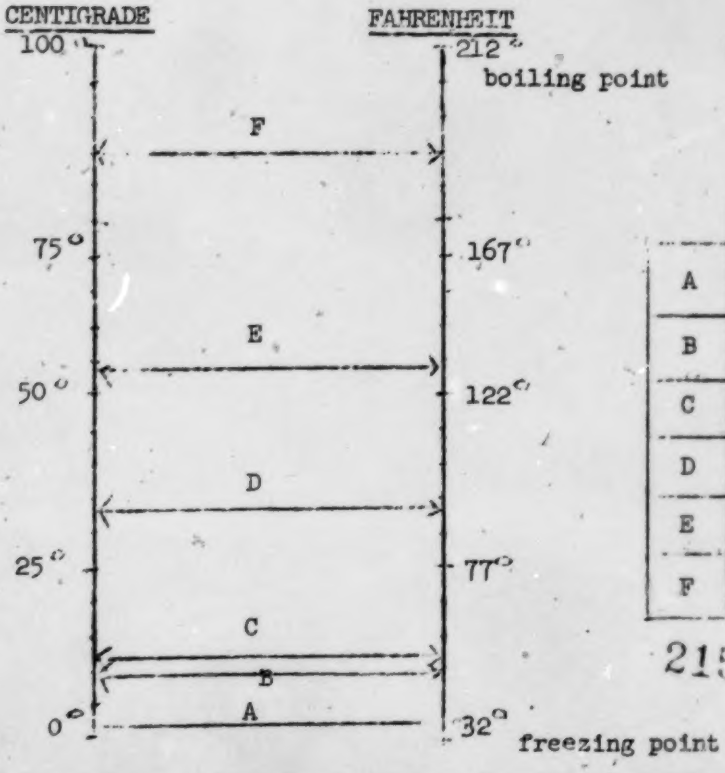
ACTIVITY 2: The student will fill in the attached table.

Suggested materials:

Worksheet and pencil

Directions to student:

Look at the attached printed scales. Five various levels B through F are designated by arrows. The answer for level A has been tabled for you. Round off the measures B through F and complete the table.

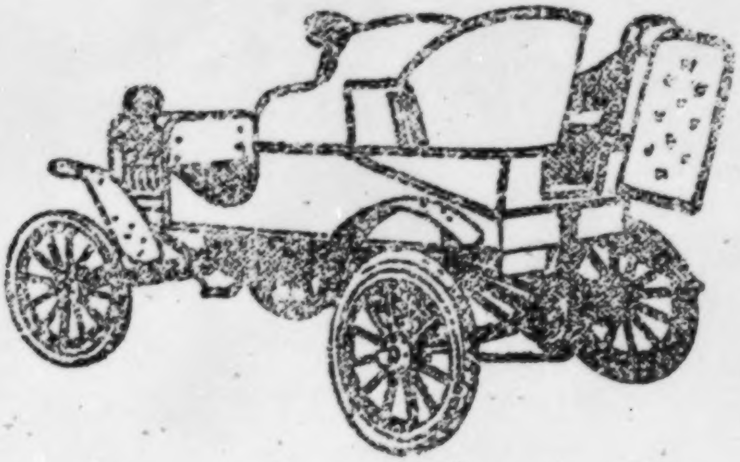


Complete the chart.

	Centigrade	Fahrenheit
A	0°	32°
B		
C		
D		
E		
F		

DIRECTIONS: *Round the population of the following towns and cities to nearest hundreds.*

1. Le Mars ____________
2. Marion ____________
3. Mount Ayr ____________
4. Clinton ____________
5. Osage ____________
6. Mt. Vernon ____________
7. Madrid ____________
8. Villisca ____________
9. Williamsburg ____________
10. Sheldon ____________
11. Hartley ____________
12. Burlington ____________
13. Armstrong ____________
14. Avoca ____________
15. Muscatine ____________
16. Des Moines ____________
17. Rockwell City ____________
18. Harlan ____________
19. West Branch ____________
20. Dunlap ____________



1. Sumner ____________
2. Rolfe ____________
3. Mount Ayr ____________
4. Clinton ____________
5. Arispe ____________
6. Libertyville ____________
7. Centerville ____________
8. Morning Sun ____________
9. Deaksville ____________
10. Missouri Valley ____________
11. New Sharon ____________
12. Cedar Falls ____________
13. Britt ____________
14. Oakland ____________
15. Sanborn ____________
16. Tipton ____________
17. Winterset ____________
18. Rock Rapids ____________
19. Adel ____________
20. Spencer ____________

Suggested strategies:

The teacher could develop the formula $F = \frac{9}{5} C + 32$ by finding the ratio between the distance of 180° on the Fahrenheit scale and 100° on the centigrade ($\frac{180}{100}$).

There is a corresponding ratio for the intervals shown in the illustration on page 161.

$$\frac{180}{100} = \frac{45}{25} = \frac{9}{5}$$

The ratio of centigrade to Fahrenheit is $\frac{100}{180}$ for the entire distance from freezing to boiling points and $\frac{25}{45}$ or $\frac{5}{9}$ for the intervals shown which gives the formula:

$$C = \frac{5}{9} (F - 32)$$

COMPETENCY: Predict the possibility of simple events occurring.

OBJECTIVE: Through the use of simple devices the student should demonstrate and predict the probability of simple events occurring.

ACTIVITY 1: Using coins to determine probability

Suggested materials:

1. Pennies or other coins to be differentiated from one another (enough for every 2 students).
2. Transparencies for teacher to show direction of activities. (For this activity see worksheet A.)
3. Small baby food jars with lids or other containers that can be used for a shaker.

Directions to student:

1. Shake 2 coins in container and read and record results (heads or tails) of coins in container. Do this 100 times.
2. Shake 3 coins in container and read and record results (heads or tails) of coins in container. Do this 100 times.
3. Find probability of results of 1 and 2 above.

Suggested strategies: (for teacher)

1. Suggest that the teacher should define probability as

$$\text{PROBABILITY} = \frac{\text{Successful tries}}{\text{Total Tries}} = \frac{\text{S.T.}}{\text{T.T.}}$$

2. Stress that the collection of this statistical data is necessary to predict probability.
3. Direct students in analyzing data from their experimentation and determine probability.
4. Direct students to analyze the statistical data obtained from their experiments. Also stress the mathematical aspect and de-emphasize the idea of gambling.
5. Have students work in pairs.

ACTIVITY 2: Using dice to determine probability

Suggested materials:

1. dice (pair for every 2 students)
2. A set of large dice for teacher visual aid
3. Transparencies for teacher to show direction of activities. (For this activity see worksheets B and C)

Directions to student:

1. Roll single die fifty times on top of the desk and record results.
2. Find the probability of results.
3. Roll a pair of dice 100 times on top of desk and record the results.
4. Find the probability of results.

Suggested strategies: (for teacher)

1. Suggest that the teacher should define probability as:

$$\text{PROBABILITY} = \frac{\text{Successful tries}}{\text{Total tries}} = \frac{\text{S.T.}}{\text{T.T.}}$$

2. Stress that the collection of this statistical data is necessary to predict probability.
3. Direct students in analyzing data from their experimentation and determine probability.
4. Direct students to analyze the statistical data obtained from their experiments. Also stress the mathematical aspect and de-emphasize the idea of gambling.
5. Teach students how to analyze the game of dice (large dice and transparencies suggested for this instruction).
6. Have the students work in pairs.

ACTIVITY 3: Using a deck of playing cards to determine probability

Suggested materials:

1. Decks of playing cards (enough for a deck for every 2 students.)
2. Transparencies for the teacher to show direction of activities.

Directions to student:

1. With a deck of playing cards, predict the probability of drawing:
   1. a spade
   2. an ace
   3. an eight of hearts
   4. a black card
   5. a black ten

Suggested strategies: (for teacher)

1. Suggest that the teacher should define probability as:

$$\text{PROBABILITY} = \frac{\text{Successful tries}}{\text{Total tries}} = \frac{\text{S.T.}}{\text{T.T.}}$$

2. Stress that the collection of lthis statistical data is necessary to predict probability.

3. Direct the students in analyzing data from their experimentation and determine probability
4. Direct the students to analyze the statistical data obtained from their experiments. Also stress the mathematical aspect and de-emphasize the idea of gambling.
5. Consider the above procedures as a pattern for further experimentation with the deck of playing cards.
6. Have students work in pairs

ACTIVITY 4: Using a set of colored objects to determine probability

Suggested materials:

1. Three sets of different colored balls or different colored objects and containers to be used for random selection (enough for every 2 students). The container should be opaque and capable of handling the sample objects.
2. Transparencies for the teacher to show direction of activities.

Directions to student:

1. With a container of 4 black objects, 5 white objects, and 3 red objects, predict the probability of drawing: (considering replacement of each object after it is drawn)
    1. One red object
    2. One white object
    3. One black object
    4. One white and one red object
    5. One white, one black, and one red object

Suggested strategies: (for teacher)

1. Suggest that the teacher should define probability as:

$$\text{PROBABILITY} = \frac{\text{Successful tries}}{\text{Total tries}} = \frac{\text{S.T.}}{\text{T.T.}}$$

2. Stress that the collection of this statistical data is necessary to predict probability.
3. Direct the students in analyzing data from their experimentation and determine the probability
4. Direct the students to analyze the statistical data obtained from their experiments. Also stress the mathematical aspect and de-emphasize the idea of gambling.
5. Consider the above procedures as a pattern for further experimentation with the colored objects.
6. Have the students work in pairs.

Worksheet A

Possible outcomes when shaking two coins in a container:

H H T T

H T H T

Question that may be asked:

1. What is the probability that two tails will occur?
2. What is the probability that one head will occur?
3. What is the probability that at least one tail will occur?
4. What is the probability that one or more heads will occur?

Possible outcomes when shaking three coins in a container:

H H H T T T H T

H H T T H T T H

H T T T H H H T

Questions that may be asked:

1. What is the probability that two heads will occur?
2. What is the probability that at least two tails will occur?
3. What is the probability that two or more heads will occur?
4. What is the probability that at least three tails will occur?

Worksheet B

The Possible arrangements coming up in one roll of two dice

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

The sum of the faces of the two dice

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Worksheet C

The sum of the faces of the two dice

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The probability of outcome

Number (outcome)	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	2	3	4	5	6	5	4	3	2	1
P (N) Probability	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

In the game of dice----on the first roll, if you get a 7 or 11 you win; if you roll a 2, 3, or 12 you loose. Should you roll any other number (4, 5, 6, 8, 9, 10) on the first roll, that number is known as your "point."

You then continue to roll the dice until you come up with your "point" or the number 7. If you roll your "point" before you roll a 7, you win, however, if the 7 comes up before your "point," you loose.

COMPETENCY: First steps in business arithmetic. Determining the weekly and biweekly wages.

OBJECTIVES: Given the hourly rate and the number of hours worked during a week, the student should be able to determine the weekly and biweekly wages.

ACTIVITY 1:

Suggested Materials:
1. Multiplication tables
2. Blank "Payroll Checks"
3. Play Money
4. Teacher created problems
5. Student workbook Page 1
6. Student homework sheet

Directions to Students:
1. Discuss with your classmates the job opportunities around your area.
2. Talk about government (civil service) and non-government (private firms) jobs.
   Suggestions: A. Payroll periods B. Minimum wage
   Note: When you earn an hourly wage, you are paid only for the hours you worked. A salary is a fixed amount, usually yearly, that is paid a person to do a job to the best of his ability. A teacher is paid a salary.
3. To find the weekly earnings, multiply the hourly rate by the number of hours of work during one week.
   Look at this example:
   Anvorak worked 15 hours in one week. Her hourly rate is $1.25. Find her weekly earnings.
   Observe: ($1.25) hourly rate x (15) hours worked.
   Solve: $1.25
   x 15
   625
   125
   $18.75 Thus: Weekly earnings: $18.75
4. Look at this example:
   Find the biweekly earnings of a person who works 30 hours a week using the following hourly rates.
   a. $2.84 b. $3.05

   Reminder: bi means two. So biweekly means happening once every two weeks. To find the biweekly earnings, we find the weekly earnings first. Then we multiply the weekly earnings by 2 (2 weeks) to find the biweekly earnings.

   Observe:
   a. ($2.84) hourly rate x (30) hours of work in one week.
   Solve: $2.84 (hourly rate)
   x 30 (hours of work during one week)
   $85.20 (weekly earnings)

Now we take $85.20 (weekly earnings) and multiply it by 2 to find the biweekly earnings.

$$\begin{array}{r} \$85.20 \\ \underline{\times \quad 2} \\ \$170.40 \end{array}$$

(biweekly earnings)

5. By following the above example, find the biweekly earnings for Part B, Example 4.

6. Take workbook pages 1 and 2 and follow instructions.

7. Check your answers by exchanging paper with your neighboring classmate or a friend.

8. If you have difficulty, raise your hand and ask your teacher or any student who understands the concept.

9. When you finish and have your answers corrected, then start working on your homework.

Suggested Strategies:

1. Solve a problem on the chalkboard and explain how to compute the weekly and biweekly earnings.
2. Make up problems at a different level of difficulty for the students to further reinforce the learning concept.
3. Try to get to each student and to help each one individually.
4. Check the students' answers by sending volunteers to solve problems on the chalkboard. Let the class correct the student at the chalkboard.
5. The students will be more interested in what they are doing if they understand why they are doing it. Discussion Questions: a. Why do people work? b. Is it interesting to compute your own weekly and biweekly earnings?

NAME ________________________________

Go over the sample problem with your teacher.

SAMPLE PROBLEM: Find the weekly earnings.
Hours worked --- 28
Hourly wage ---- $1.80

To find the weekly wages, we multiply the hourly rate by the number of hours of work. So, we multiply ($1.80), the hourly rate by (28) hours of work. Thus:

```
  $1.80
  x  28
 ------
   1440
   360
 ------
 $50.40  Weekly earnings
```

Now solve these problems. Find the weekly earnings of a person who works:

1. 35 hours at $2.30 an hour.

2. 31 hours at $3.05 an hour.

3. 39 hours at $4.10 an hour.

4. 331/3 hours at $2.84 an hour.

5. 37 1/2 hours at $3.00 an hour.

6. If a person works 40 hours per week, what is his weekly pay at each of the following hourly rates:

a. $2.50 a. ______________ b. $1.95 b. ______________
c. $3.80 c. ______________ d. $5.65 d. ______________
e. $4.75 e. ______________

7. If a person works 30 hours weekly, what are his biweekly earnings at each of the following hourly rates.

a. $1.80 a. __________________
b. $3.50 b. __________________
c. $2.65 c. __________________
d. $4.20 d. __________________
e. $2.45 e. __________________

Student Homeword Sheet #67

NAME: ______________________________

1. List several occupations around your area in which wages are paid on an hourly basis

    a.

    b.

    c.

    d.

2. How often do government employees get paid?

3. Write a paragraph explaining each of these terms: Overtime, time-and-a-half, double time.

COMPETENCY: Use adders and/or calculators to solve addition, subtraction, multiplication, and division problems.

OBJECTIVE: To be able to add a series of one, two, three, or more digit numbers with an adding machine.

ACTIVITY 1:

Suggested materials:
Monroe-Victor adding machine for each student or group.

Directions to the student:

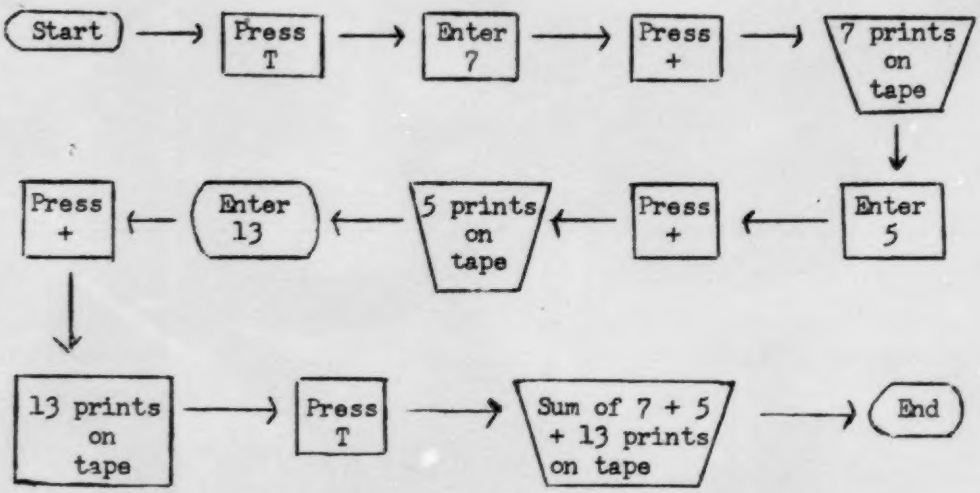
Adding:

1. Beginning to add a series of numbers with an adding machine, always clear the machine first by pressing the T (total) key.

2. Now in adding a series of numbers, press the appropriate keys for the first addend and upon completion of this task, you then must press the key on the adding machine which represents the operation you are doing, which is addition. Therefore, press the + (plus) key. One will continue to repeat the same operation with the machine until you have recorded all addends in the given problem.

3. After recording all addends your next task is to find the sum. Therefore after recording the last addend and pressing + (plus) key, you then press T (total key) and the result will be the sum of the series of numbers you recorded in the machine.

Example: Addition
7 + 5 + 13 = ____


Start
Press T
Enter 7
Press +
7 prints on tape
Enter 5
Press +
5 prints on tape
Enter 13
Press +
13 prints on tape
Press T
Sum of 7 + 5 + 13 prints on tape
End

Suggested strategies:

A worksheet could be given to students on adding whole numbers.

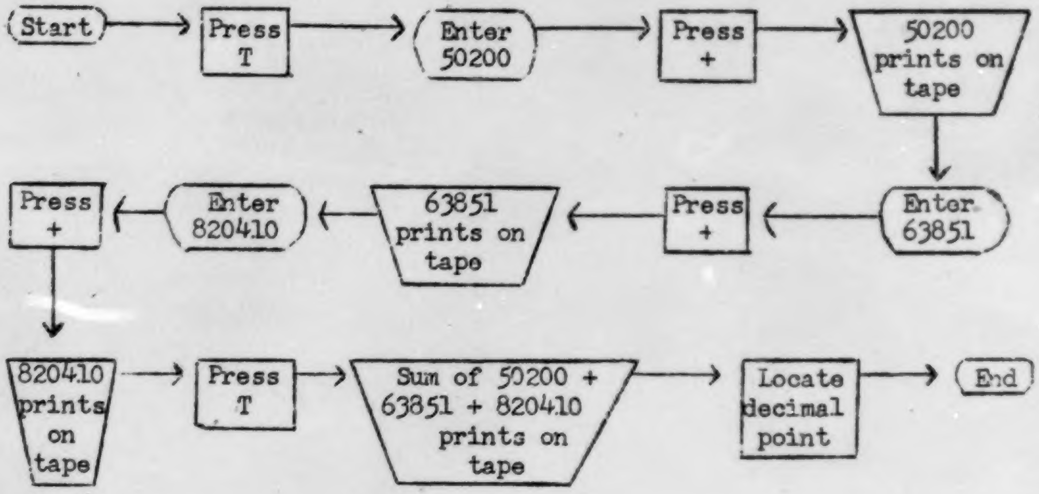
OBJECTIVE: To be able to add decimal numerals.

Directions to the student:

Remember to find the decimal numeral with the greatest amount of digits to the right of decimal point to determine how to position your numbers on adding machine.

Example: 5.02 + 6.3851 + 82.041 = ________

The greatest amount of digits to the right of the decimal point in the example is four (appearing in the numeral 6.3851). Therefore, one must always maintain four digits in each numeral one enters to the right of the decimal. For example, the first numeral 502 must be entered as 50200. Please take note that you do not enter a decimal point on an adding machine; you must do this upon totaling your answer.


Start
Press T
Enter 50200
Press +
50200 prints on tape
Enter 63851
Press +
63851 prints on tape
Enter 820410
Press +
820410 prints on tape
Press T
Sum of 50200 + 63851 + 820410 prints on tape
Locate decimal point
End

Suggested strategies:

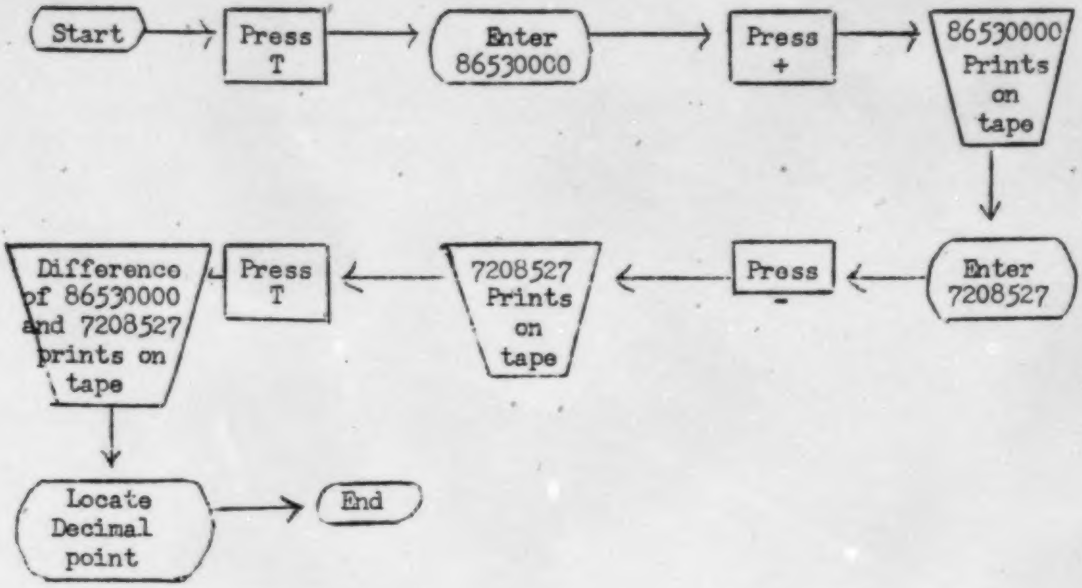
Give decimal numerals to add, also arrange these numerals vertically and horizontally.

OBJECTIVE: To be able to subtract whole numbers and decimal numerals.

Directions to the student:

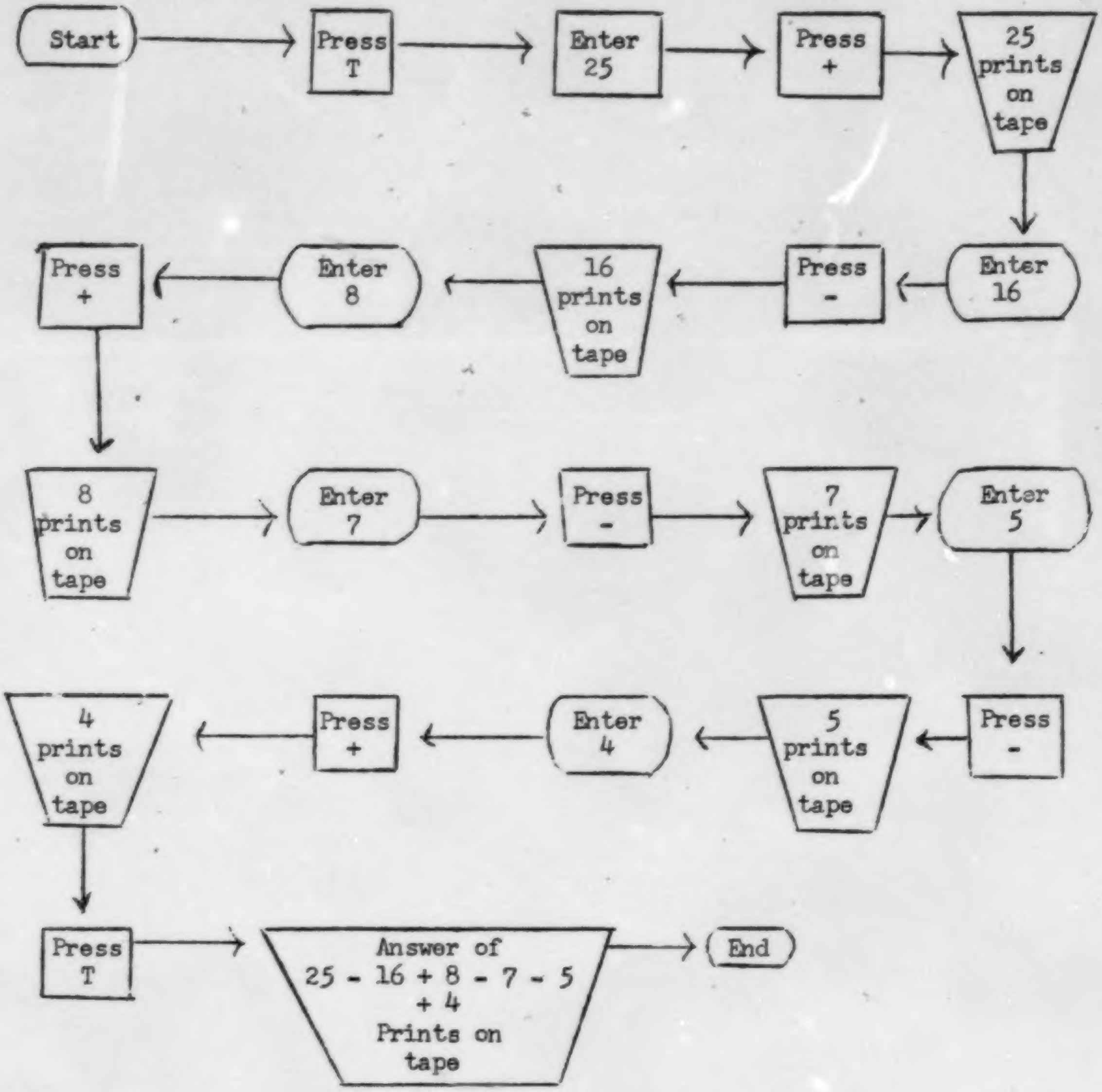
865.3 - 72.08527 = ________

Please refer to Activity 2 for directions on handling decimals for addition as same principles apply in subtraction.


Start
Press T
Enter 86530000
Press +
86530000 Prints on tape
Enter 7208527
Press -
7208527 Prints on tape
Press T
Difference of 86530000 and 7208527 prints on tape
Locate Decimal point
End

Example 2: Series of combined addition and subtraction

25 - 16 + 8 - 7 - 5 + 4 = ________


Start
Press T
Enter 25
Press +
25 prints on tape
Enter 16
Press -
16 prints on tape
Enter 8
Press +
8 prints on tape
Enter 7
Press -
7 prints on tape
Enter 5
Press -
5 prints on tape
Enter 4
Press +
4 prints on tape
Press T
Answer of 25 - 16 + 8 - 7 - 5 + 4 Prints on tape
End

Suggested strategies:

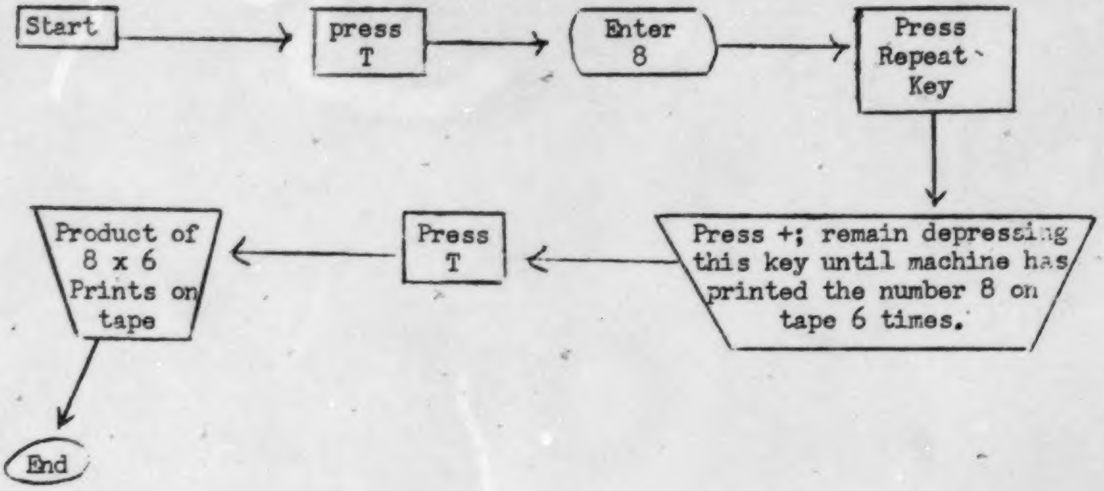
Worksheets could be made up on the following:

1. spending money
2. maintaining a checking account
3. budget
4. verbal problems which the student will read and determine whether to press the + or - key after entering number.
5. units of measure (English as well as metric)

OBJECTIVE: To be able to multiply, where multiplier is a one digit numeral with the adding machine.

Directions to the student:

8 x 6 = ________


Start
press T
Enter 8
Press Repeat Key
Press +; remain depressing this key until machine has printed the number 8 on tape 6 times.
Press T
Product of 8 x 6 Prints on tape
End

Suggested strategies:

1. Multiplier should always be one digit
2. Work with costs of items. Make a chart where the students multiply then add totals.
3. Work with invoices

OBJECTIVE: To be able to multiply two, three, or more digit numbers with adding machine

Directions to the student:

Example: 45 x 86 = ________

One could enter 45 in the machine, then press the repeat key and then depress the ÷ key until 86 "45's" appear on the tape. This, however, would require too much time, so let's analyze how one can multiply with the machine with fewer operations.

45 x 86
45 x (80 + 6)
(45 x 80) + (45 x 6)
(45 x 10 x 8) + (45 x 6)
(45 x 10) x 8 + (45 x 6)
(450 x 8) + (45 x 6)
45 x 86 is the same as (450 x 8) + (45 x 6)

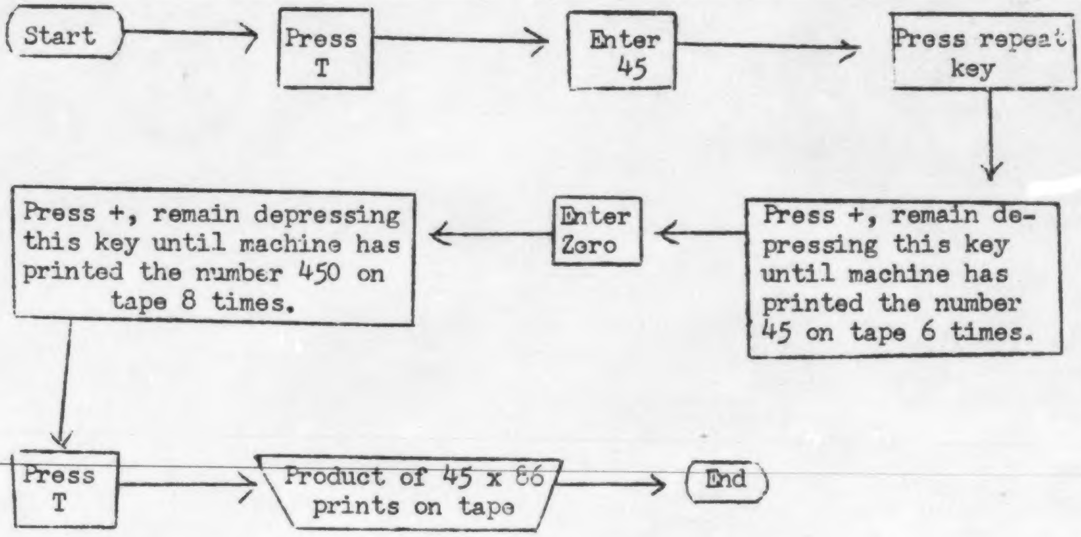
Operation A: 45 x 6 =

Enter 45, press the repeat key, then press the + key, remain depressing this key until 6 "45's" print on the tape.

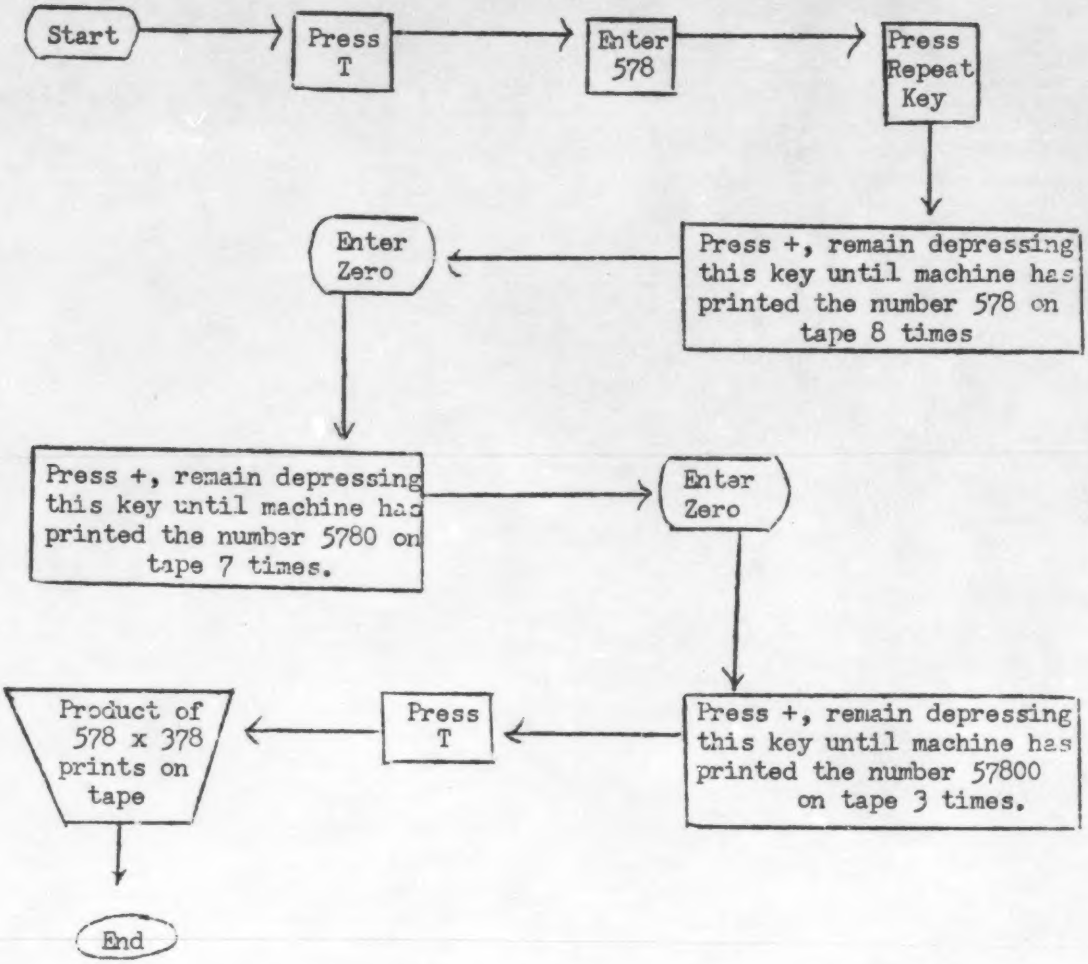
Operation B: 450 x 8 =

How can I enter 450 in the machine? Your machine still has a memory "45", so enter a zero and now the machine has 450 in memory. I then press the + key (you still have to repeat key depress) and remain depressing this key until 8 "450's" print on the tape. Then press the total key; the product of 45 x 86 prints on the tape.

45 x 86 = ________

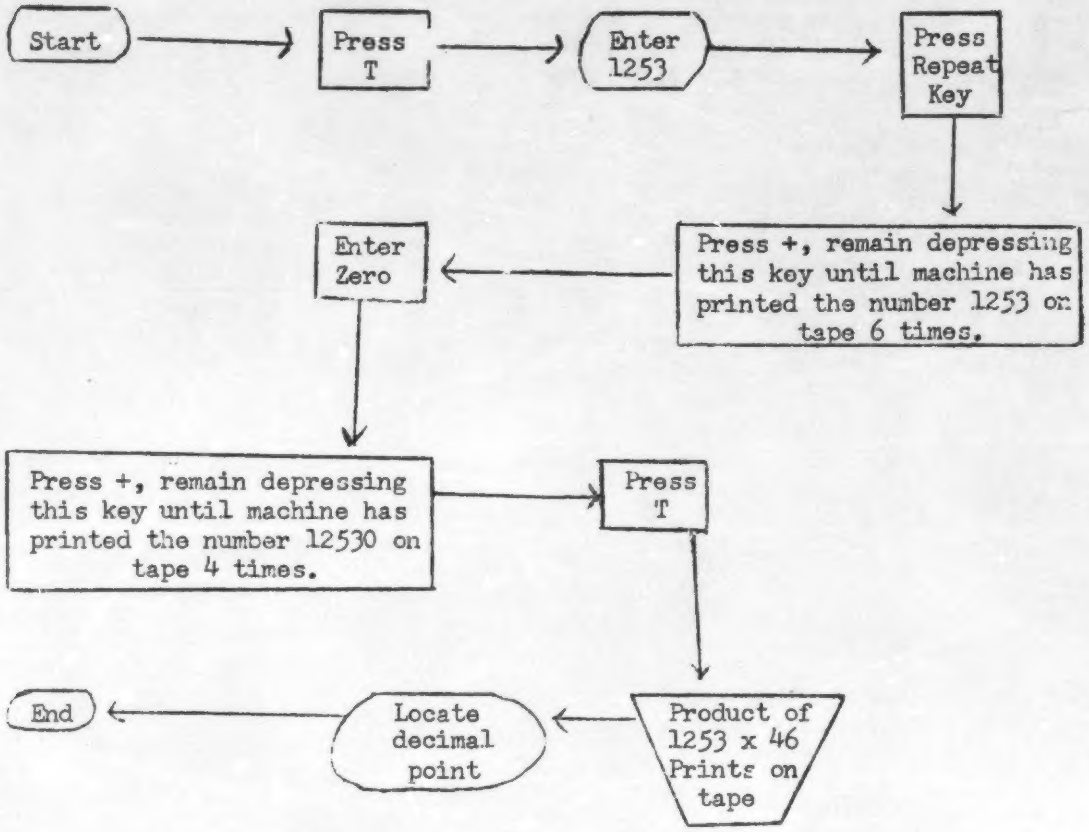

Start
Press T
Enter 45
Press repeat key
Press +, remain depressing this key until machine has printed the number 45 on tape 6 times.
Enter Zero
Press +, remain depressing this key until machine has printed the number 450 on tape 8 times.
Press T
Product of 45 x 86 prints on tape
End

Multiply 578 x 378 = ________


Start
Press T
Enter 578
Press Repeat Key
Press +, remain depressing this key until machine has printed the number 578 on tape 8 times
Enter Zero
Press +, remain depressing this key until machine has printed the number 5780 on tape 7 times.
Enter Zero
Press +, remain depressing this key until machine has printed the number 57800 on tape 3 times.
Press T
Product of 578 x 378 prints on tape
End

Example 2:
46% of 1253 is ________

Rewrite into arithmetic sentence; it becomes
1253 x .46 = ________


Start
Press T
Enter 1253
Press Repeat Key
Press +, remain depressing this key until machine has printed the number 1253 on tape 6 times.
Enter Zero
Press +, remain depressing this key until machine has printed the number 12530 on tape 4 times.
Press T
Product of 1253 x 46 Prints on tape
Locate decimal point
End

Suggested strategies:
Worksheets could be made up of the following items.
1. whole numbers
2. decimal numbers--payrolls
3. percentages-sales tax; retirement; social security; interest; investments.
4. payrolls--covering the above three items.

COMPETENCY: Write simple sentences showing the relations =, <, >, for two given numbers.

OBJECTIVES: The student who successfully completes this unit should be able to:

1. Supply the missing relations to make given number sentences true.
2. Supply the missing element or number to make given number sentences containing the relations true.
3. Supply the missing operation to make given number sentences true.
4. Determine whether sentences containg =, <, >, or their negations are true or false.

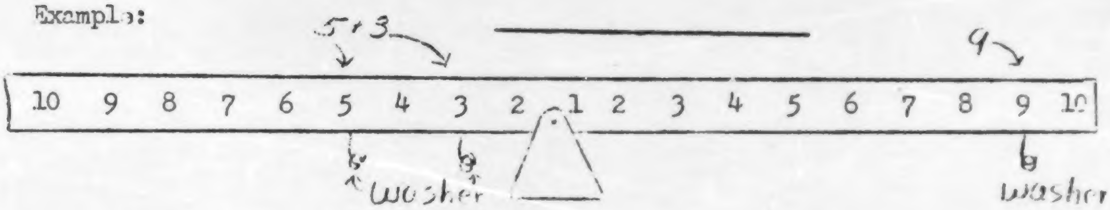
To achieve the above objectives, the following activities are proposed:

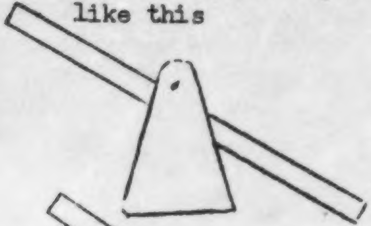
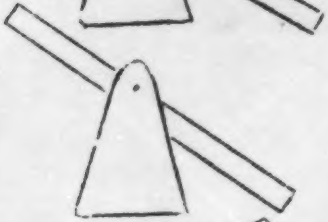
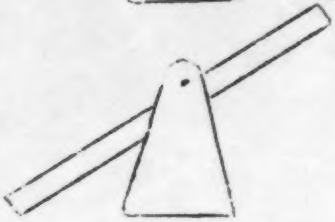
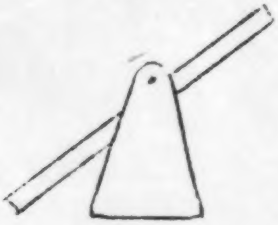
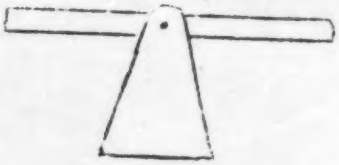
ACTIVITY 1:

Suggested materials:
One beam balance per student, washers.

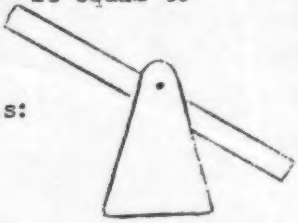
Directions to the student:
1-A. Fill in the blanks below with <, >, =, $\nleq$, or $\ngeq$. Place washers on the hooks on the left side of the balance beam for the numbers on the left side of the blanks and washers on the hooks on the right side for those on the right.

Example:



If the balance tips like this	You may place the following in the blank	Which means
	$<$	is less than
	$\ngeq$	is not greater than or equal to
	$>$	is greater than
	$\nleq$	is not less than or equal to
	$=$	is equal to

Example: $3 + 4$ ________ 9 Balance tips like this:



Therefore: Means:

$3 + 4 < 9$ is less than

or $3 + 4 \ngeq 9$ is not greater than or equal to

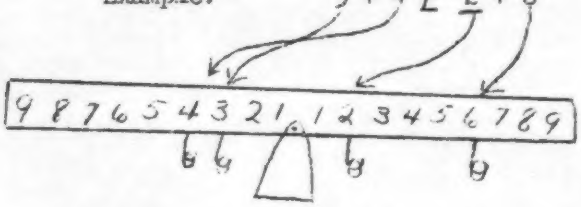
Problem	Means
1. 7 ________ 5	1. ________
2. 4 ________ 8	2. ________
3. 5 + 4 ________ 7	3. ________
4. 8 + 3 ________ 9 + 2	4. ________
5. 2 + 5 ________ 8	5. ________
6. 2 + 3 + 4 ________ 10	6. ________
7. 9 + 3 + 2 ________ 8 + 4 + 5	7. ________
8. 3 x 4 ________ 2 x 5	8. ________
9. 2 x 7 + 5 ________ 4 x 4 + 2	9. ________
10. 1 + 2 + 3 x 4 ___ 3 x 5	10. ________

ACTIVITY 1-B:

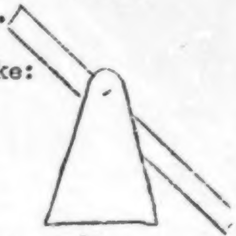
Directions to the student:

Fill in the blanks below with whole numbers.
Place washers on the hooks on the left side of the beam balance for the numbers on the left side of the relation (<, >, =, ≰, ≱) and washers on the right side for those on the right. Use the smallest possible number.

Example: 3 + 4 ≱ 2 + 6



Balance must look like:



Note: 3, 4, etc. also work, but 2 is the smallest number.

1. $2 + 3 < 1 + \_\_\_\_\_\_\_\_$

2. $7 + 3 = \_\_\_\_\_ + 4$

3. $9 + 3 \nleq 5 + \_\_\_\_\_\_\_\_$

Note: $\leq$ means less than or equal to.

4. $7 + 5 \leq \_\_\_\_\_ + 6$

5. $5 \times 3 \nleq \_\_\_\_\_ \times 8$

Fill in the blanks below with the largest possible whole number that will make the sentence true.

6. $5 + 7 > \_\_\_\_\_ + 2$

7. $3 + 10 \nleq 5 + \_\_\_\_\_\_\_\_$

Note: $\geq$ means greater than or equal to.

8. $6 + 5 \geq 3 + \_\_\_\_\_\_\_\_$

9. $5 + 3 \nleq 7 \times \_\_\_\_\_\_\_\_$

10. $5 \times 2 + 4 > 3 \times \_\_\_\_\_ + 6$

Fill in the blanks below with an operation symbol (+, -, ÷, x) which will make the sentence true.

11. $3 \_\_\_\_\_ 4 \nleq 9$

12. $7 \_\_\_\_\_ 5 < 6$

13. $8 \_\_\_\_\_ 2 \nleq 5$

14. $7 \_\_\_\_\_ 3 > 9 \_\_\_\_\_ 2$

15. $8 \_\_\_\_\_ 4 \nless 5 \_\_\_\_\_ 3$

Suggested strategies:

This activity is designed for independent study and could be used for review or reinforcement. Problems can be made much more difficult. Prior work with a beam balance is desirable. Beam balances can be easily made using rulers with nails placed at the numbers. Do not make them too sensitive. If the beam balance is new to the student, allow him time to "mess around" with it.

ACTIVITY 2: Possession
A game involving inequalities

Suggested materials:

Two decks of cards are used. One deck consists of four sets of numbers from 1 to 10. For this deck, a regular bridge deck with the jacks, queens, and kings removed may be used with the aces counting as 1. The second deck consists of 10 cards on which are printed the relations: $=$, $\neq$, $<$, $\leq$, $>$, $\geq$, $\not<$, $\not>$, $\not\leq$, $\not\geq$, respectively. Sixty toothpicks, buttons, beans, or some such objects.

Object of the game:

The first player to possess thirty toothpicks is the winner. Other ways for determining a winner might be:

1. a definite number of toothpicks other than thirty
2. a time limit where the player possessing the most toothpicks at the end of the time is the winner
3. a specific number of hands (the player possessing the most toothpicks at the end of these hands wins.)
4. when one person has all the toothpicks

Number of players: 3, 4, 5, or 6.

Rules of the Game: Version 1.

1. Evenly divide the toothpicks among the players.
2. Each player draws a playing card to determine the dealer. High card wins the deal.
3. Shuffle the 40 playing cards and 10 relation cards.
4. Place the relation cards face down in the center of the table.
5. Deal out one playing card to each of the players including the dealer.
6. Turn over a relation card.
7. The dealer then goes to each player in turn and using his own playing card as the left side of a relation, the upturned relation card, and the other players' cards as the right side, determines if the sentence is true or false.
8. If the sentence is true, the player must give the dealer one toothpick. If it is false, the dealer gives up a toothpick to the player.
9. If the dealer makes a mistake in determining whether the statement is true or false and is caught by the player he is comparing with, he must forfeit two toothpicks to that player.
10. After four deals, the deal is then passed to the player on the left, who shuffles both the playing and relation cards and continues play.
11. The relation cards may be shuffled anytime a player requests it and must be reshuffled everytime a new dealer starts.
12. If you are out of toothpicks, you are out of the game.

Rules of the Game: Version 2.

Change only number 5 as follows:

5. Deal out two playing cards to each of the players including the dealer.

Rules of the Game: Version 3.

Works well for older students and creates a lot of excitement:

Directions to the student:

1 - 6 as in Version 1.

7. After seeing his playing card, the dealer's playing card and the relation card, each player decides how many toothpicks he is willing to risk (from one to five). These he places on top of his playing card.
8. Each player including the dealer is dealt one more card, face up.
9. The dealer now proceeds from player to player as in Version 1, using the sum of the two cards.
10. If the sentence is true, the dealer picks up the toothpicks placed on the card. If it is false, he must match the number of toothpicks on the card from his own pile.

Numbers 9 - 11 in Version 1 conclude this version.

Suggested strategies:

This card game is designed to be used to review and reinforce an understanding of the equality and inequality relations. It has a lot of versatility and can be quickly adjusted to the ability of the students by adding new relations to the first deck or by replacing the natural number cards with rational numbers, irrational numbers, etc. Instead of toothpicks, points may be kept on paper for each trick won.

Hint: This game works very well with pennies.